What do we mean when we say we "want students to understand exponential growth?"

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Research reported in this presentation was supported by National Science Foundation Grant No. EHR-0353470, with Patrick Thompson as principal investigator. Any conclusions or recommendations stated here are those of the author and do not necessarily reflect official positions of NSF.

Standards for Exponential Growth

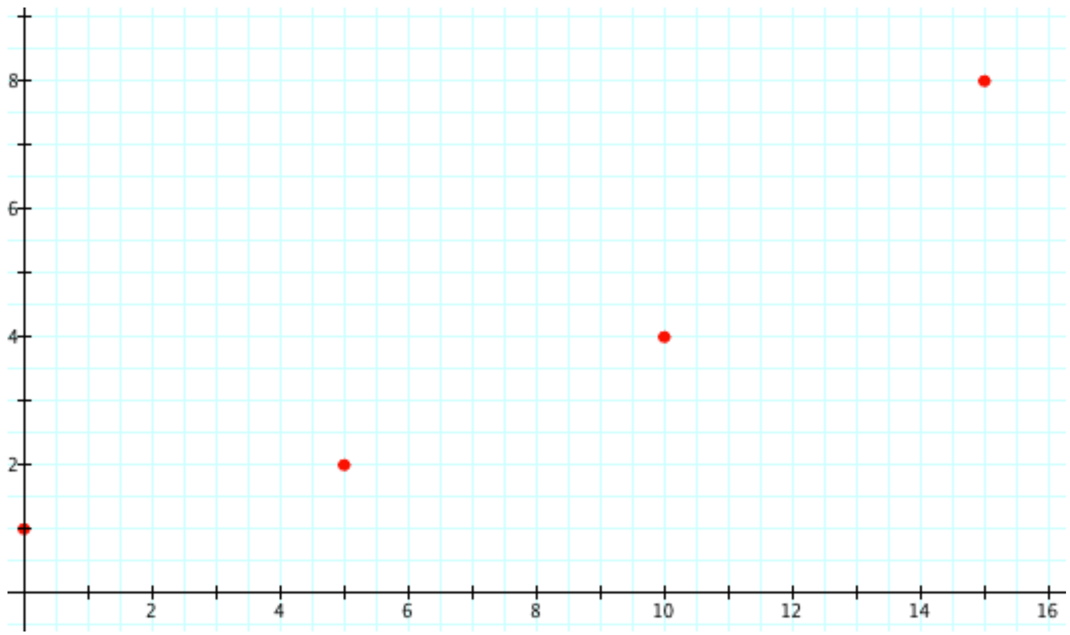
- Students in 9th-11th grades should "understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions..." (NCTM Principles and Standards)
- What properties?
  - Rate proportional to amount (Defining property)
  - Geometric growth (NCTM; Common Core)

Big ideas of this talk

- "Filling in the gaps" of geometric growth is not trivial
- There are more ways to think about exponential growth that deserve consideration

“Filling in the gaps”

Bacterial Growth: Beginning with one, doubling every 5 minutes.



How many bacteria after one minute?

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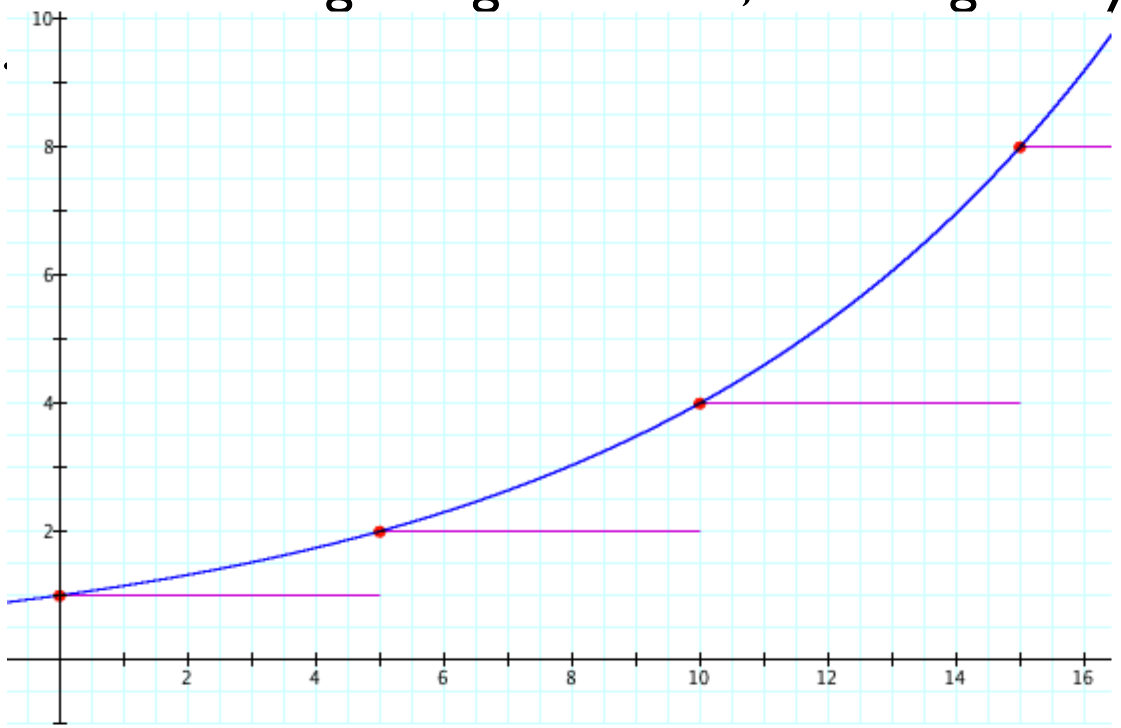

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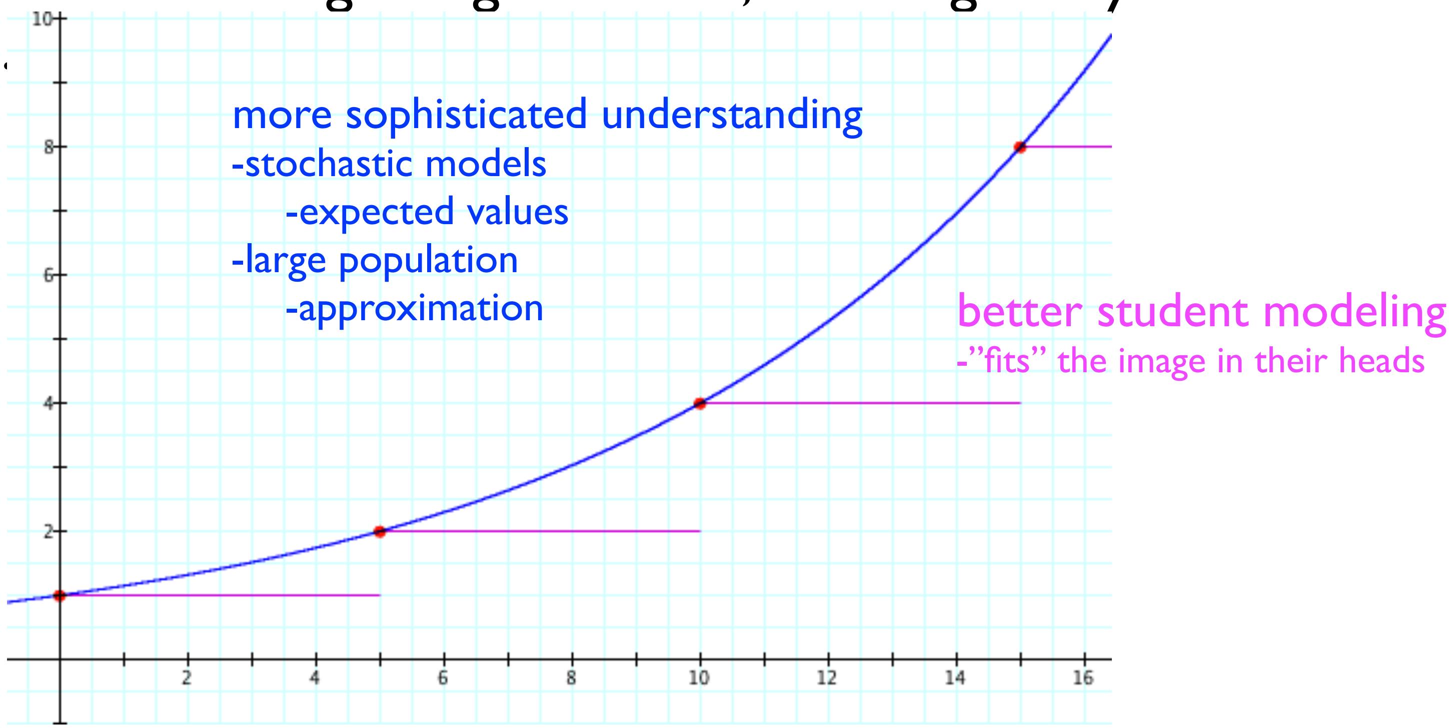


How many bacteria after one minute?

Other ways of thinking about exponential growth

- Two non-honors Algebra II students
  - Class spend a year working with ASU
    - Focus on continuous variation (in small pieces)
  - 15 teaching interviews
    - Targeted toward teaching the logistic ODE
    - Financial and biological exponential modeling
    - Students had many different understandings of exponential growth

Geometric Understanding

Jodan bank uses a simple interest policy for their EZ8 investment accounts. The value of an EZ8 account grows at a rate of eight percent of the initial inve- initial investment per year create a function that gives the value of an EZ8 account at any time

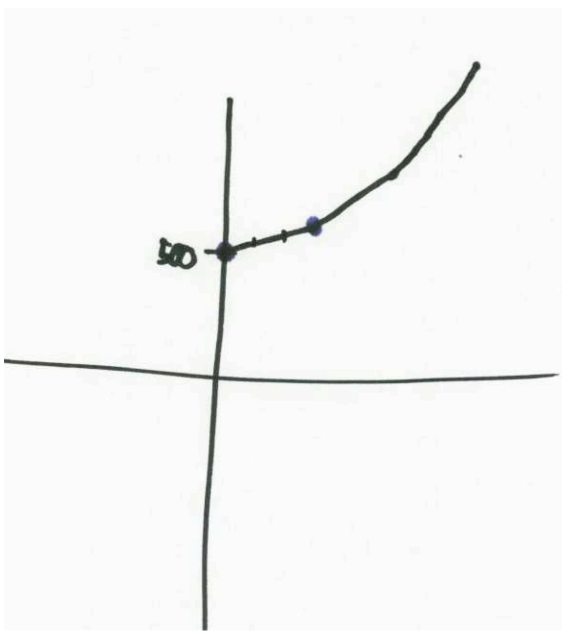
if you put in a certain amount I would say like ten dollars

in the next year she'd be like ten eighty

then the next year it's like you know, whatever eight percent of ten-eighty is

$$q(x, n) = n + .08(n)x$$

Piecewise Linear Compound Interest



$500 + .08(500)x$

beginning 1st $<$ x $<$ end 1st

$0 \le x < \frac{1}{4}$ of year

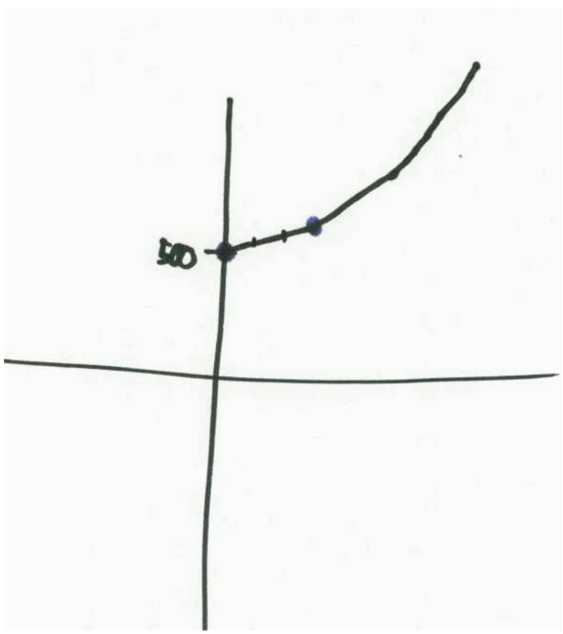
$510 + .08(510)(x - \frac{1}{4})$

$\frac{1}{4} < x < \frac{1}{2}$

$\boxed{520.2} + .08(520.2)(x - \frac{1}{2})$

$\boxed{\frac{1}{2} < x < \frac{3}{4}}$

Piecewise Linear Compound Interest



$$500\left(1+\frac{.08}{4}\right)^0 + .08(500)x$$

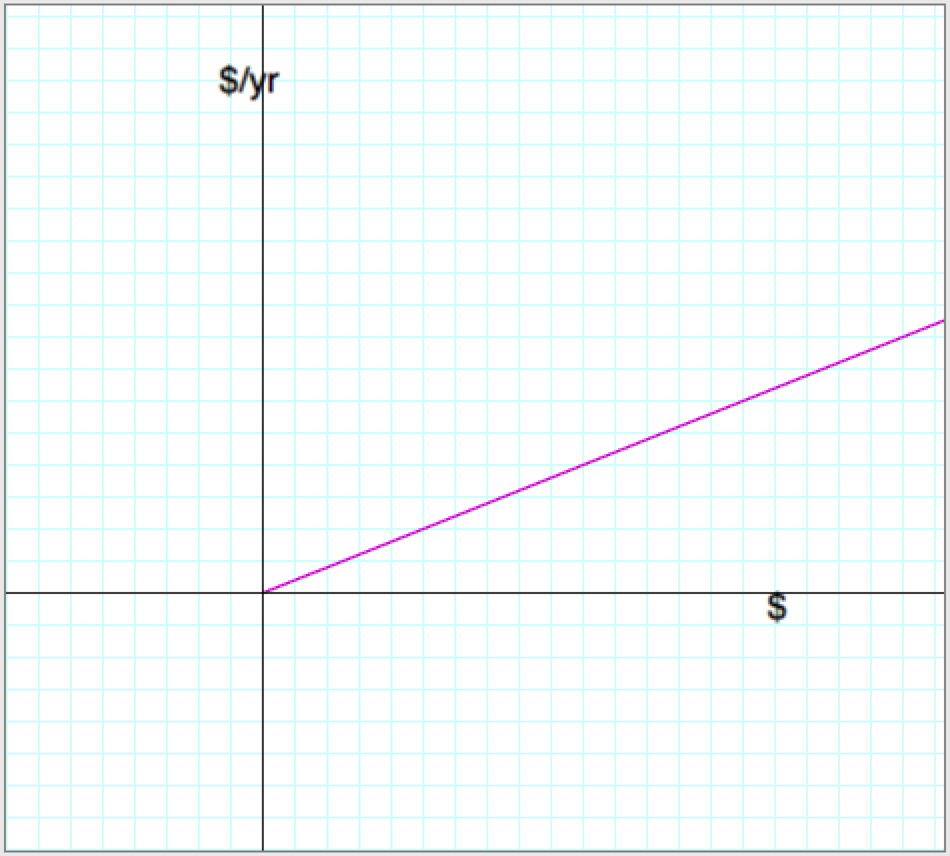
$$500\left(1+\frac{.08}{4}\right)^1 + .08(510)\left(x-\frac{1}{4}\right)$$

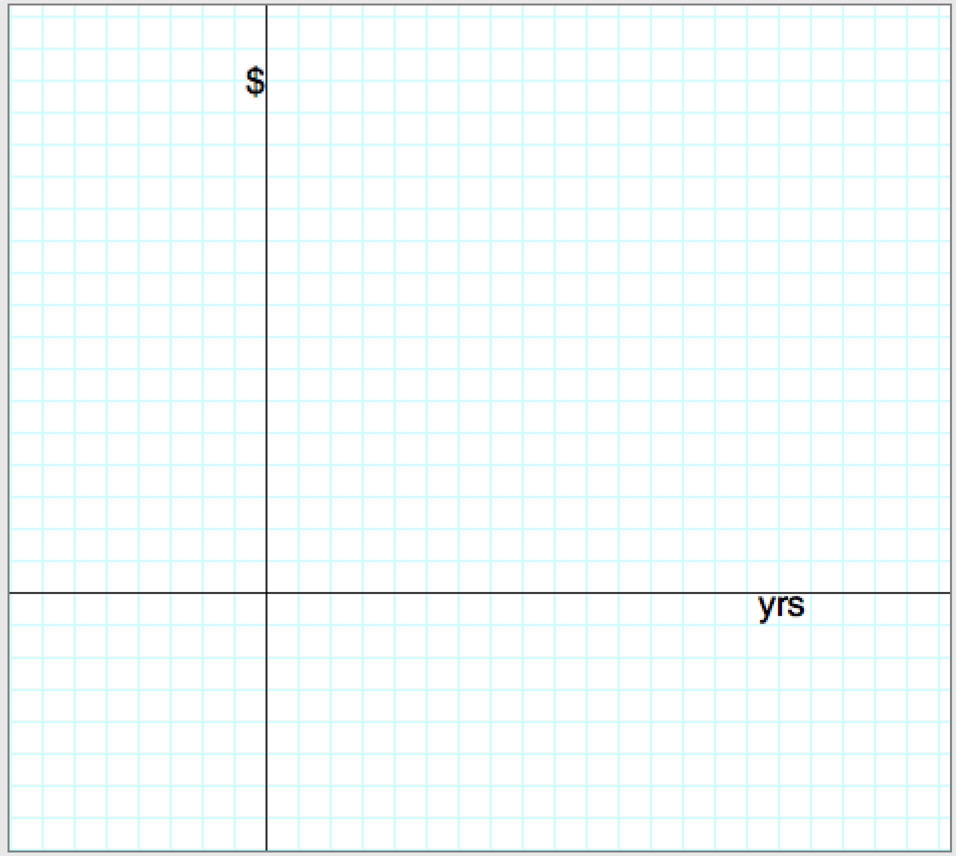
$$500\left(1+\frac{.08}{4}\right)^2 + .08(520.2)\left(x-\frac{1}{2}\right)$$

$$500\left(1+\frac{.08}{4}\right)^5 + .08\left(500\left(1+\frac{.08}{4}\right)^5\right)\left(x-\frac{5}{4}\right)$$

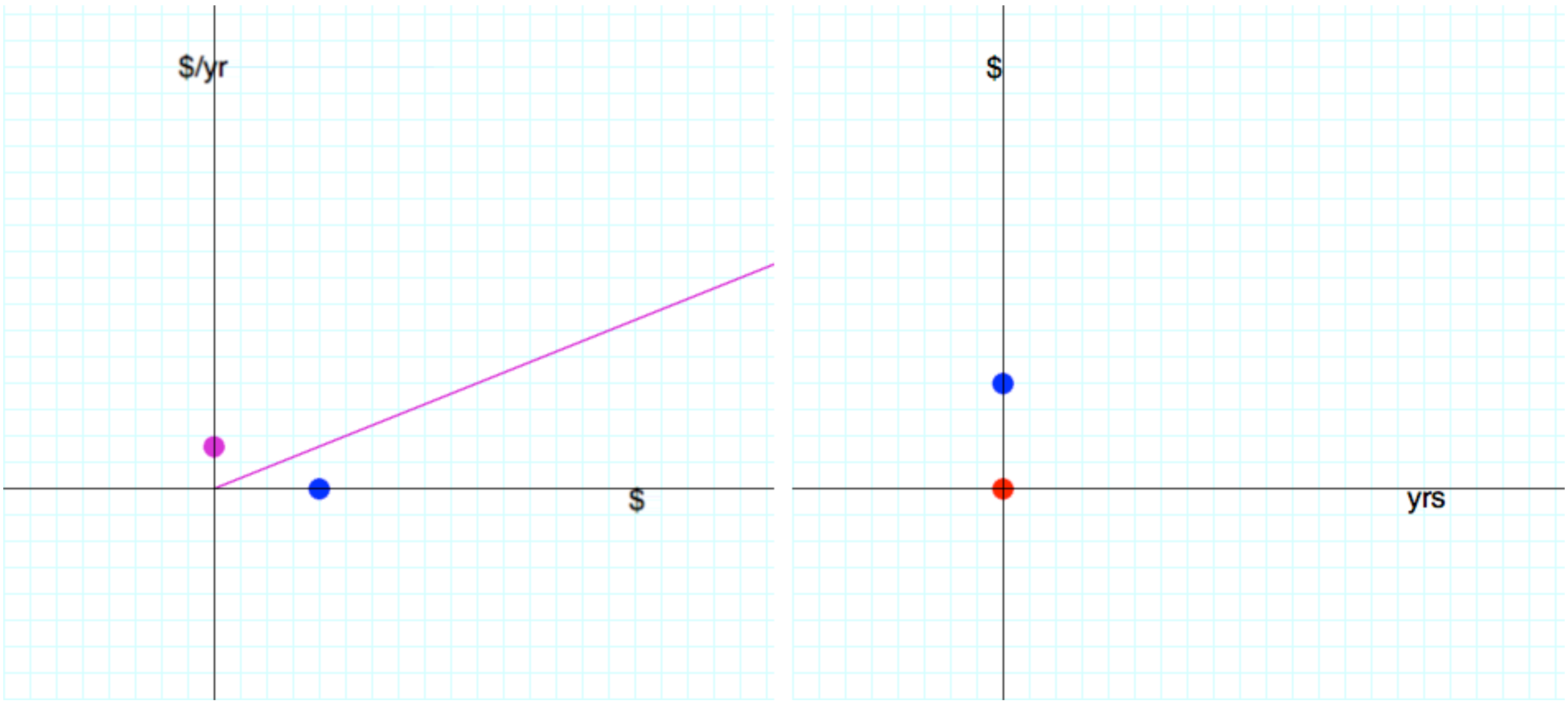
$$\frac{5}{4} < x < \frac{6}{4}$$

Phase Plane





Derek (1)



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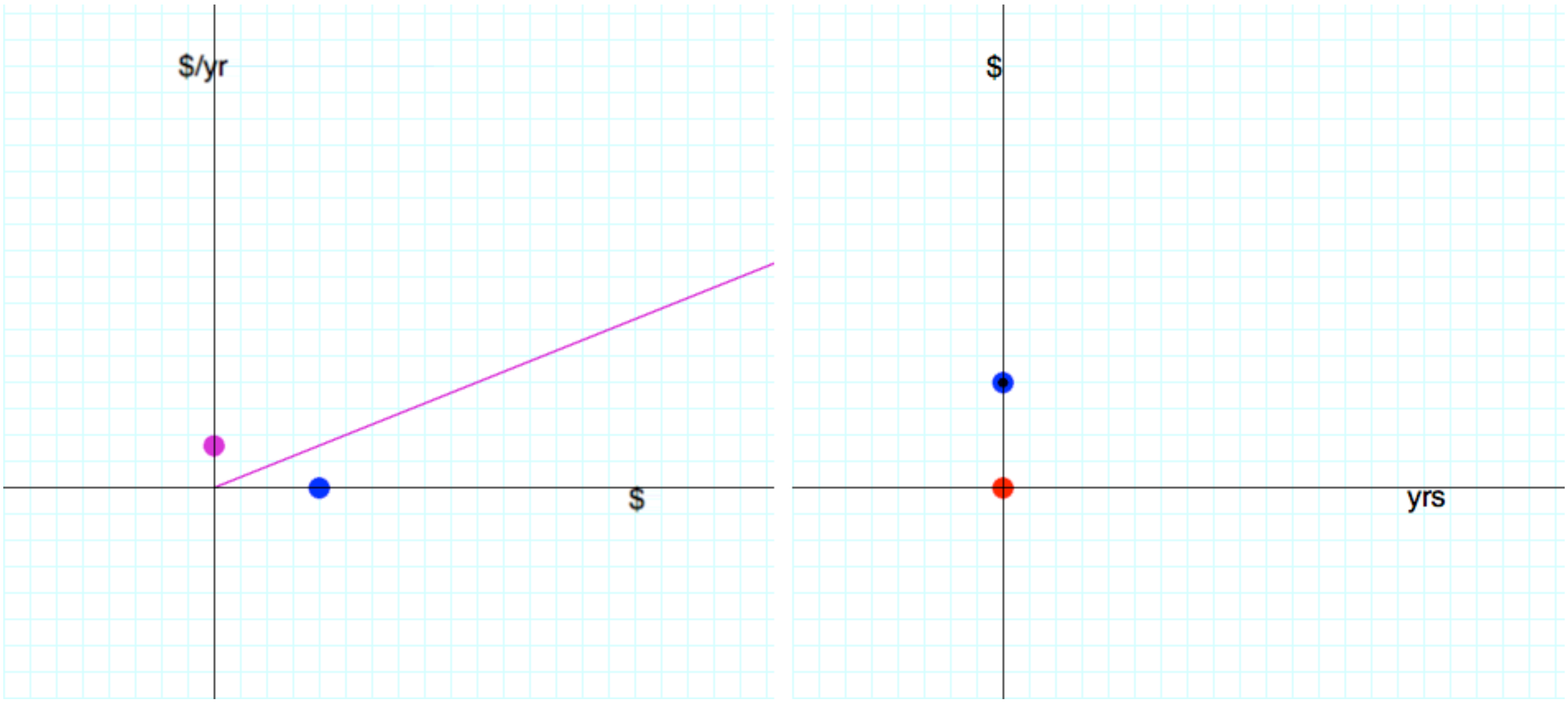

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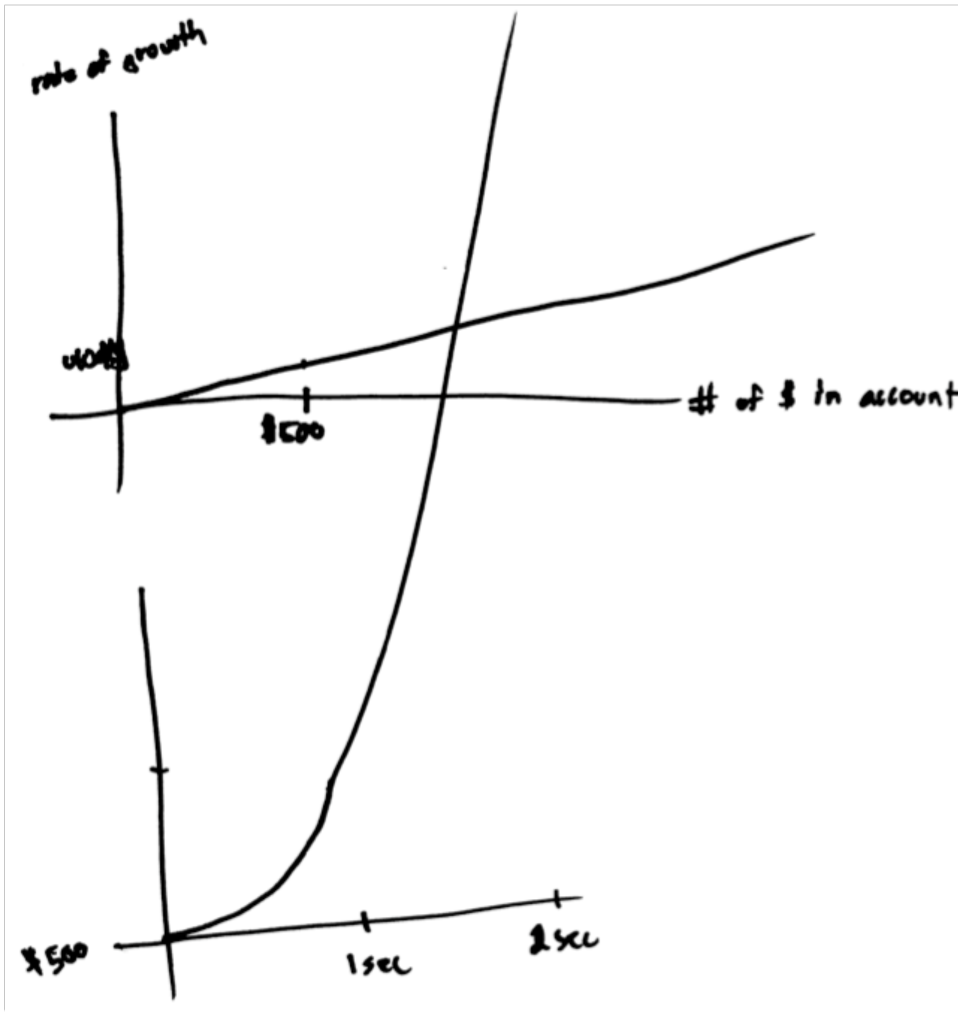


Carlos: So can you show me how the money in your- in your account is growing, umm.

Derek: On that axis?

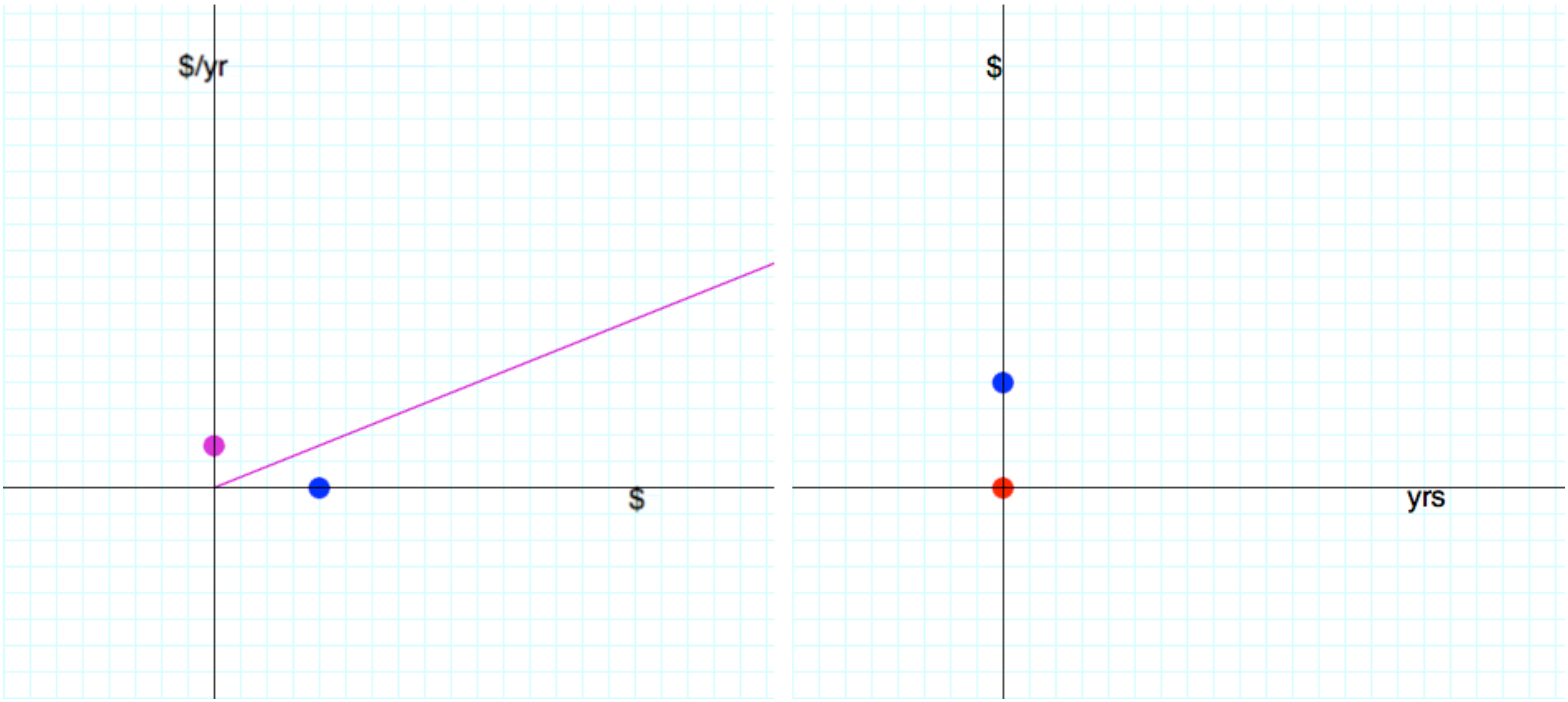
Carlos: By moving your finger along this axis, yeah.

Derek: Like starts slow and then just keeps getting faster and faster.

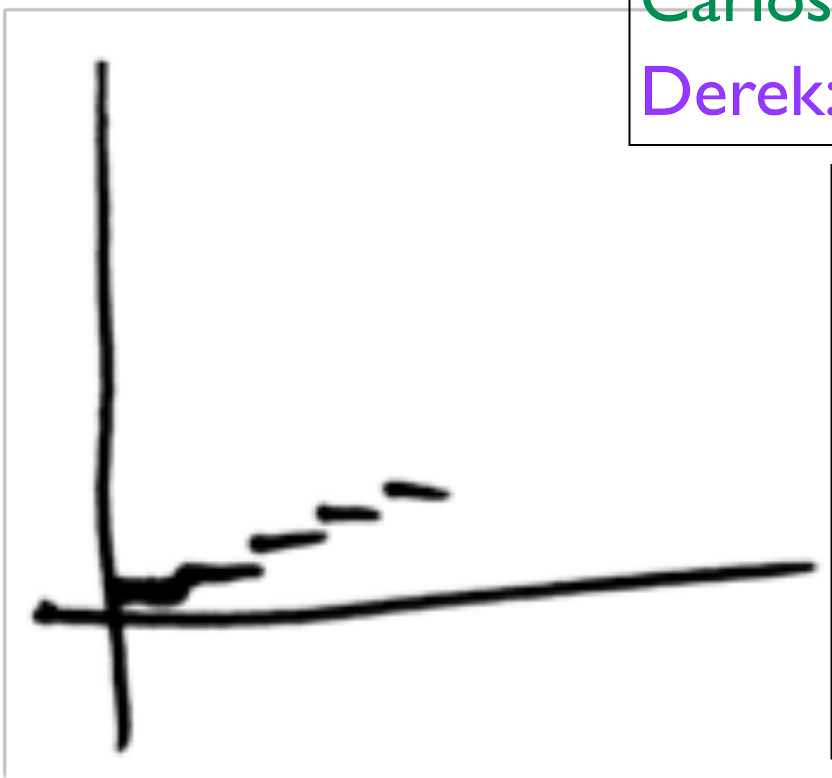
Carlos: OK, umm, and what about the rate of growth?

Derek: It would also start slow and keep getting faster and faster.

Derek (2)



Derek (2)



Carlos: What about how long these are?

Derek: They get shorter.

Carlos: So you said these are getting shorter and shorter and shorter. what about

Derek: The jumps?

Carlos: jumps?

Derek: They're staying the same, because you're jumping one by one.

Harmonic Exponential

$$\frac{1}{w} = ry$$

Time to to reach y=2 $\frac{1}{r}$

Time to to reach y=3 $\frac{1}{r} + \frac{1}{2r}$

Time to to reach y=4 $\frac{1}{r} + \frac{1}{2r} + \frac{1}{3r}$

Time to to reach y=n $\frac{1}{r}\sum_{k=1}^{n-1}\frac{1}{k}$

Summary

- There are multiple ways of imagining exponential growth
  - Geometric, Compounding, Differential, Stochastic, etc.
  - They produce different results until you take a limit.
- Derek's results were "inconsistent" because he used multiple ways of thinking with mathematically different results
  - "inconsistent" does not mean "incorrect"
  - Derek's "code switching" between multiple ways of thinking reflects professional usage.

Summary

- It is possible for students to learn very sophisticated understandings of exponential growth with very few tools (proof of concept)
- Better design could make these ways of thinking accessible to more students
- The “best” way of thinking is situation/model dependent

Questions

- What ways of thinking do we want students to learn?
- How should we tailor or curricula to teach them?

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Thank you.