A Change in one Classroom

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Abstract

This paper describes a change in one high school teacher's teaching style over a two month period. I compare two problems, one from August and one from November, and contrast the types of thinking that the teacher uses these problems to promote, by using Skemp's distinction between relational and instrumental understanding. From the comparison of the problems and the comparison of the teacher's values, I conclude that the quality of a problem is always constrained by the values of the teacher that teaches it, a position which has consequences for curriculum design.

Introduction

This paper is an account of how one teacher's image of teaching may have changed over the course of participating in the Teacher's Promoting Change Collaboratively (TPC^2) program, and in a teaching experiment affiliated with the TPC^2. The TPC^2 program consists of a specially designed two semester course, as well as Reflecting on Practice Sessions (RPSs), in which teachers meet in groups to discuss their instructional practice with the aid of a program facilitator. Details of the TPC^2 can

be found in Thompson and Carlson (2006) as well as Thompson's article in this volume.

Although the TPC^2 provided insight into how teachers think about mathematics and instruction, all of these insights occur outside of a classroom context. As the project progressed into it's second year, it became necessary to see how the teachers took their ideas and understandings from the TPC^2 into their own classrooms, if in fact, they did so at all. Making student thinking a core issue in TPC^2 discussion became a primary goal of the project; and in order to do that, we needed artifacts of student thinking.

One high school mathematics teacher ("Augusta") agreed to participate in a teaching experiment in one of her ninth-grade mathematics classrooms. The purpose of which was to provide the artifacts of student thinking described above. For this purpose, both the class and a series of interview with Augusta were videotaped. The teaching experiment was never intended to be a research project, and the researchers who collected student artifacts and taped the class also made suggestions about curriculum. A description of teaching experiment methodology can be found in (Steffe & Thompson 2000) and (Steffe 1983).

However, even though the teaching experiment was never intended to be a research project, I noticed an interesting difference in Augusta's teaching style from the beginning of the semester (August) to the end of the semester (November).

Two Problems

I will illustrate this difference by presenting two problems that Augusta used, one in August, and one in November. The August problem is a catchup word problem originally designed by Paul Foerster (1994):

Augusta: alright, ladies and gents. So. We have a scenario.. to go over and to

discuss, so read it with me as I read aloud. Robin Banks robs a bank. ha ha ha ha

Student: [laughter]

Augusta: Yeah, that was- That was on purpose. Robin Banks robs a

bank, and drives off. A short time later, he passes a truck stop at

which police officer, Willie Katchup

Student: [laughter]

Augusta: Ha ha ha- Man, we're funny! police officer Willie Katchup is

dining. Willie receives a call from his dispatcher, and takes off in

pursuit of Robin five minutes after Robin went by. Robin is driving at

60 miles per hour. Willie pursued him at 80 miles per hour.

The November problem, in contrast, is the sort of bare, context-less linear inequality that one would expect to find in a traditional high school algebra textbook:

$-4 < 2x < 10$

However, I would argue that the transition from the word-problem in August to the bare inequality in November was actually an improvement for Augusta. I will do this by arguing that it is not the problem Augusta chose that is important, but rather the

type of thinking that Augusta used that problem to promote. I will do this using Skemp's (1976, 1979) distinction between relational and instrumental understanding.

Robin Banks

When Augusta presented the Robin Banks problem, the students struggled with what, for them, was an ambiguity. One they had converted the 5 minute time gap to 0.083 hours, they had no way of deciding between the forms *80(x + 0.083)* and *80x + 0.083* as an expression to represent the distance Willie had traveled. That is, the students understood from previous lessons the rule that "distance equals rate times time," but in this situation where there was more than one time, they didn't know which time to use.

When Augusta discussed this issue in an interview with Pat, Pat suggested that she present the problem as a temporal frame of reference problem, that from Robin's frame of reference, I can predict everything that is going on in Willie's frame of reference, without having to step into Willie's frame of reference. Together, Pat and Augusta constructed a lesson plan around this idea.

> Pat: ... So you can And that's the key for really unraveling it. It's a
> frame of reference problem. .... So. How are you going to adjust?
> Augusta: [laugh] That's what I was just thinking about. I think I'm just
> gonna have to have like, on the board or something, you know, a
> diagram of each of their [thunder] a diagram
> Pat: This is supposed to happen at night.
> Augusta: I know! of umm- their own individual clock and talk about

how, you know, what we just talked about. Where I'm just gonna have

to go through with them that yes, they have individual clocks, and

actually like chart them on the board.

Pat: Yeah

Augusta: And then say, but you know, I don't have to necessarily

know what they're both doing at the same time cause I know that no

matter what Willie left five minutes later. So, Robin's clock says this. I

know where Willie's clock is.

Pat: Instead of calling it a clock, call it a stopwatch.

Augusta: stopwatch. kay. And just talk about that, you know, if I'm

able to, you know, find out or keep track of umm the time that's

elapsed on Robin's, you know, stopwatch, I don't need to go find

Willie and ask him what his stopwatch says, I know what his stopwatch

says. And I'll even probably even do it both ways.

Augusta also came up with an alternative lesson plan, and in the end, it was the

second lesson plan, which did not touch on temporal frame of reference, that she

chose (details can be found in the transcription in the appendix). When students

continued to have difficulties with the parentheses, this is the explanation that she

finally settled on giving to the students:

Augusta: It's a number of hours. Kay? So think about this. If you

don't put parentheses to group Your units of time. Then you're taking

a distance and subtract a number of hours. Does that make sense?

Can you 25 feet and say 25 feet minus one hour?

Augusta had available a relational explanation for this problem (the temporal frame of reference), but instead what she chose to give to the students was another rule: You can't take a time away a distance. This method of patching a difficulty in one rule ("d=rt") with another rule("You can't take a time away from a distance") is characteristic of Skemp's (1979) instrumental learning.

Furthermore, the rule that she has given the students is post-hoc, in that it provides a way of checking one's answer for a specific mistake, but it does not provide a method for reaching that answer. For example, the system of rules described above does not explain why it is unreasonable to say that the distance Robin has traveled is *60x*, and the distance that Willie has traveled is *80x*.

Inequalities

Two months later, after numerous interviews and researcher interventions of the type described above and in the Appendix, Augusta presented a very different problem in a very different way: *-4 < 2x < 10.* In this example, the student does not have difficulty solving the problem, but Augusta is not satisfied with the correct answer. She wants the student to think of the answer in a particular way:

Augusta: So there's the original problem negative four is less than or equal to two x is less than or equal to ten. Tell us what you did.

Student: Umm... I divided by two on everything.

Augusta: kay.

Student: And then I got negative two is less than or equal to x is less than or equal to five ... and the, I did filled in circle on negative two and then a filled in circle on five? And I connected them?

Augusta: Ok, so again. I mean, she divided everything by two because the x is being multiplied and that's fine. But think about it. The interval you need to show me isn't really between -4 and 10. The interval that you- that you're showing me When It's Doubled. When it's doubled is in between -4 and 10. So you're actually between -2 and 5. Cause when you double -2 and 5 you're at -4 and 10. That's exactly where you needed to be. And so, she she shaded in between but we should probably check, just to make sure.

In this example, Augusta is asking the student to think of the problem in a particular way: by reasoning about the transformative relationship between the interval that the student has been given, and the interval that the student wishes to find. In this case, that the interval *(-4,10)* is twice as large as the interval *x.* This is a case of Augusta promoting a relational understanding, in that this manner of thinking not only helps the student to choose to divide by two, but explains *why* dividing by two works. This emphasis on *why* is characteristic of Skemp's relational understanding.

In Contrast

In August, Augusta chose to patch the student's rule of "d=rt" with another rule: "You can't take a time away from a distance", even though an alternative was available, indicating that Augusta valued instrumental understanding over relational understanding.

In contrast, in November, Augusta chose to emphasize a particular way of thinking: "Think about the relationship between the interval you have and the interval you want." even though the student has arrived at the correct answer by an instrumental action "divide everything by two." In November, Augusta shows that she is valuing a relational understanding over an instrumental understanding.

Unfortunately, because the teaching experiment was not originally designed to be a research project, many of the planning sessions with Augusta were not videotaped. So it is impossible to pinpoint the chain of ideas that led to this change, other than the fairly vague explanation that over a lengthy period of time, Augusta had be exposed to and planned her class with researchers who valued a relational understanding.

Consequences

The consequence of this for the researchers in the experiment is that the second semester of the class could proceed with a clearer focus, in that now that Augusta valued a relational understanding, it became possible to design with her, a class that focused on teaching a few general mathematical ideas rather than a large number of mathematical rules.

However, a larger implication is that addresses curriculum design. The August problem was clearly a better designed problem than the November problem, but Augusta used those problems very differently. This suggests that even the best designed curriculum will always be constrained by the values of the teacher that teaches it. In other words, it's not the problem that's important. It's the teacher.

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Appendix: Transcriptions

08-24 Robin Banks Class

Augusta: alright, ladies and gents. So. We have a scenario.. to go over and to discuss, so read it with me as I read aloud. Robin Banks robs a bank. ha ha ha ha
Student: [laughter]
Augusta: Yeah, that was- That was on purpose. Robin Banks robs a bank, and drives off. A short time later, he passes a truck stop at which police officer, Willie Katchup
Student: [laughter]
Augusta: Ha ha ha- Man, we're funny! police officer Willie Katchup is dining. Willie receives a call from his dispatcher, and takes off in pursuit of Robin five minutes after Robin went by. Robin is driving at 60 miles per hour. Willie pursued him at 80 miles per hour.
Student: Does pursued mean caught?
Augusta: Pursuit means-
Student: Followed
Augusta: Followed. Went after. yep.
    So if you're in pursuit of somebody, you're going after them. pretty much, ok? So. That's you're scenario.
    Now, couple things I wanna talk about. First of all, the moral of the story is I wanna know when Willie actually catches up to Robin. Kay? But. I had to anticipate a couple things, because I know how you guys work. If I would have just said, "When does he catch up?" You would have played with numbers upon numbers upon numbers until it worked. Cause you guys know what it means to 'work' at this point. And you would have not shown me anything profound as far as expressions or equations or the need to define variables, or all that stuff that we've been working on. kay? So I will tell you right now. Don't just play with numbers. If you read the very first kind of question, so to speak it says "Write an expression that represents how far apart Robin and Willie are at any point in time." kay? So I-I'm immediately trying to get through to you guys. I do want expressions. We are still defining variables. We are still practicing representing situations, kay? we're modeling represent-rr modeling situations with expressions and equations. kay?
    Umm. So. Lessee. Couple things we should discuss. First of all, if you gonna know how far apart Robin and Willie are what are you gonna need to know? .... Mr... [Student] Sorry, [Other Student], I know your hand was up, I'm gonna pick on [Student] If I wanna know how far apart Robin and Willie are at any point in time what are somet things I'm gonna need to know about them?
Student: How far.. like Robin is?
Augusta: Kay. So I need to know how far just Robin himself is. Kay, what else? umm.... [Student].

What else am I gonna need to know in order find out how far apart they are?

Student: How fast they're going?

Augusta: Ok, is that given to me?

Student: m-hm

Augusta: It is, so Robin is 60 miles an hour. I'm sure that will come into play. kay? got some speeds involved. [Student]?

Student: How far Willie is?

Augusta: Ok. How far Willie is. .... Good. Good. So if I know how far Willie is at any point in time. and I know how far Robin is at any point in time can I figure out how far apart they are?

Student: ...yeah.

Augusta: yeah? how? [Student], how? ...Robin's the bad guy, Willie's the cop.

Student: how far apart they are?

Augusta: Uh huh, if I knew where Robin was at some point in time, and if I knew where Willie was at some point in time How would I know how far apart they were?

Student: five miles, so if he's going 6, or other one's going 60 and then he starts chasing him 5 minutes after he passed where he was

Augusta: Ok, ok, but don't give me a specific example of where you think they'll- how apart you'll think they'll be, but just in general how do you find out how far apart two things are assuming you know where they each are on their own. so if I'm here and [Student]'s there, how far apart are we? see what I'm sayin'? How do you figure that out?

Student: umm... [inaudible]

Augusta: Think back to Whirlwind and Kim and Allison, and everything. When we were figuring out how far apart they were, what did we have to think about?

Student: subtraction? I dunno. [laughter] I don't

Augusta: Do you wanna use a lifeline?

Student: yeah

Augusta: How about? [Student].

Student: Umm... subtraction?

Augusta: Kay. Their difference.

Student: #1 Subtraction. Their difference. You did? #

Augusta: #2 Oh, I- I'm glad ta [inaudible]. I said subtraction. yeah. # Did you say subtraction?

Student: #1 Ah, doesn't. No I said subtraction. #

Augusta: #2 I thought you said "fraction." # I'm sorry. I thought you said "fraction." I heard fraction. yes. subtraction. You need their difference. kay? And we did this before. And I don't want to give you too much 'cause I want you to do a lot on your own. But if you know where Robin is, and you know where Willie is, how far apart they are is their difference. So I'll let you play with that, you know, in a minute.

But there's one last thing we should touch base on before I cut you loose. Read it again with me like kinda the last portion of it. Willie recieves a call and takes off in pursuit. Five. Minutes. After Robin went by. Robin's going 60 miles per hourrrr.. And Willies's going 80 miles per hourr.. Do you think it's an issue that we're talking five minutes vs. miles per hour? Minutes? Hours?

Student: yeah. [Mumbling]

Augusta: If I'm travling 60 miles per hour. And I'm only on the road for five minutes. Would you do 60 times 5?

Student: no.

Augusta: No. It's be 60 times 5 if I was on the road for five-

Student: hours

Augusta: -hours. I'm not on the road for five hours. Five minutes. So is this an issue?

Student: 12... one twelfth?

Augusta: Where'd you get one twelth?

Student: cause there a five twelfths in an hour.

Augusta: How do you know there's five twel-

Student: I mean there is - I meant there is 12 twelfths in an hour? wait...

Student B: Twelve fives?

Student: Yeah! twelve fives! yeah yeah.
Augusta: So there's five minutes
Student: And so it would be one tweflth of 60 miles
Augusta: 60 minutes is how many hours?
Student: one..
Augusta: Just one of them. So five minutes what [Student] was starting to say is like five sixtieth's- he divided sixty into five equal parts, which is twelve. and so .. He said that this is one twelfth of an hour. What do you guys think about that?
Student: yeah.
Augusta: [Student], you agree?
Student: Yeah.
Augusta: You sure?
Student: Yeah... [inaudible]
Student: [cough]I was wrong.
Augusta: No, you're not wrong. You just got there so darn quick, I had to make sure everybody else was with ya. kay? So, we do need to to be talking about you know- hours. If we're talking miles per hour We need to be in hours. Now. I'll ask you guys for your opinion. Do you wanna use one twelfth? Or would you be more comfortable using a decimal?
Student: decimal
Augusta: Decimal. How do you turn one twelfth into a decimal?

08-24 Robin Banks - Post Class Interview

Pat: So how did you feel about.. the lesson?
Augusta: mm... organized chaos. ...
Pat: well, aside from that. ...
Augusta: I mean... rgh! I just don't feel like they're there. Will they ever be there? .... ehh. ... I thought going over the homework went well. .... they seemed to be really aware of kinda what stuff represented. and, you know. They were saying it pretty accurately. And you know, in a way that is very meaningful. So I thought the homework went well..
Pat: Now. Do you think that the minutes thing might have they were at a fragile place with regard to turnign thing-turning the process over to them, right?
Augusta: mhmm
Pat: Do you think the minute thing was maybe just a little too much complexity?
Augusta: I don't even think it was the minute thing. Know what I think it was? I think it was the fact that were were now adding on or taking away a time. when the problems that they had done over the past two days were adding on, or taking away a distance.
Pat: Ok, so what's the significance? I mean, adding is adding.
Augusta: Right, but they I think they were just used to the fact, where once they has their 60x or their 80x. 'Cause a lot of them got there. They could say that if Robin's travling at 60 miles an hour then he's gone 60 x. Where x is the number of hours since he robbed the bank.
Pat: Right.
Augusta: Like that I saw a lot of. And I saw a lot of 80x. .. And then they changed x to be x is the number of hours since Willie left.
Pat: Right.
Augusta: And then they were like so do I just add five or, "point oh eight three?" and I'm like... deeew.. you just added a time to a distance. You know, cause the expression 80x is a distance. And they were like, "Yeah?"
Pat: Isn't that what they did? They did all their adding to a distance?
Augusta: In the past yes, but today no. They needed to add their extra to the time.
Pat: Umm.. ... Is there a different issue going on here? because... Willie's clock is behind... see. What-What they've been used to doing is is saying is how fast you go
Augusta: mhmm

Pat: For how long you go.
Augusta: mhmm
Pat: Okay? You look at your clock.
Augusta: mhmm.
Pat: Okay? So the thing is Willie needs to be looking at .. uh Willie has a clock. .... let's see how can I [inaudible]? See there's a shift in frame of reference.
Augusta: mhmm
Pat: Because you ask Willie "How long have you been going?" well. Umm... Willie's going 80? It's 80 times the number of hours that this says I've been going.
Augusta: mhmm
Pat: [cough] and looking at his clock. And Robin - If you ask Robin, how far have you gone? Well... it's 60 miles per hour times how ever much time has gone by on my clock.
Augusta: How are their clocks related?
Pat: Well... no. I mean, that's not the issue. We don't You don't. I don't ask you how your watch is related to my watch.
Augusta: mhmm
Pat: But the- but the situation is now we are observers, and we're only looking at one clock. we're making inferences about the clock that we can't see. Say that we're looking at Robin's clock. Ok, well, Willie left and Robin's clock says he's been on the road for 10 minutes. ... Well do we have to go now over to Willie's clock to see how long he's been on the road? See what I mean? So It's -uh-it's Getting an amount of time someone-elapsed time for someone else based on the elapsed time you know for this person.
Augusta: yeah.
Pat: ... and so there's that shift. You have to do that shift before you even think about
Augusta: mhmm
Pat: how far they've gone.
Augusta: mhmm
Pat: It's a- It's a- .. This is a classic frame of reference problem.
Augusta: ....yeah ..And that's just it. That's what they weren't seeing. They were looking from each clock individually.
Pat: Right.
Augusta: And they weren't looking at, you know, setting a frame of reference - somebody's clock - and understanding that they can still talk about what the other one is doing at that same point of reference in time. ...
Pat: Not just what they're doing.
Augusta: How far they've gone
Pat: But you can talk about what ... you- By knowing something in this frame of reference You can conclude how it looks in the other frame of reference.
Augusta: mhmm
Pat: So you're seeing how much time as elapsed in this frame of reference. By knowing that. You can know how much time has elapsed in this frame of reference.
Augusta: mhmm
Pat: ... So you can And that's the key for really unraveling it. It's a frame of reference problem. .... So. How are you going to adjust?
Augusta: [laugh] That's what I was just thinking about. I think I'm just gonna have to have like, on the board or soemthing, you know, a diagram of each of their [thunder] a diagram
Pat: This is supposed to happen at night.
Augusta: I know! of umm- their own individual clock and talk about how, you know, what we just talked about. Where I'm just gonna have to go through with them that yes, they have indivudual clocks, and actually like chart them on the board.
Pat: Yeah
Augusta: And then say, but you know, I don't have to necessarily know what they're both doing at the same time cause I know that no matter what Willie left five minutes later. So, Robin's clock says this. I know where Willie's clock is.

Pat: Instead of calling it a clock, call it a stopwatch.

Augusta: stopwatch. kay. And just talk about that, you know, if I'm able to, you know, find out or keep track of umm the time that's elapsed on Robin's, you know, stopwatch, I don't need to go find Willie and ask him what his stopwatch says, I know what his stopwatch says. And I'll even probably even do it both ways.

Pat: See and that's- ok that just occured to me. That seems like that's really key.

Augusta: mhmm

Pat: Stop talking about a clock. Start talking about a stopwatch. Because it's all abotu elapsed time.

Augusta: mhmm ... ...yeah. So I mean I think I'm just gonna have to go over just that whole idea.

Pat: mhmm

Augusta: And then.. you know, hopefully come to the correct versions of the expressions. ..and then..

Pat: Yeah, see, they're not used to making the adju-these frame of reference adjustments.

Augusta: mhmm

Pat: Well, they-they're not used to it. They haven't seen it.

Augusta: mhmm ... [thunder] Sorry, I'm like a little kid. I'm like intrigued by this. I wanna go outside and look. I'm gonna spend a good chunk of the hour doing this, so. And I-I'm sure I'll address on even more issues that I'm thinking of right now. but now I'm thinking what do I wanna do for them to have over the weekend. What do I want them to have over the weekend. ...I want them to have more word problems hehe ..

Pat: Uh, well you. well. You-you're trying to get away from scaffolding.

Augusta: mhm

Pat: umm. How about if I make up some I mean this is just- I-I'll try- I'll send you some travel notes.

Augusta: mkay.

Pat: ah- frame of reference problems. Just adjusting for one frame of reference.

Augusta: Ok.

Pat: Were, where that's the focus. ...

Augusta: yeah

Pat: Here's some information about, you know, Billie's situation

Augusta: mhmm

Pat: I mean, here's the general scenario: here's some information about Billie's situation. What can you infer about Sally's situation?

Augusta: mhmm ... yeah ...so what I would like their test to be over on Tuesday is I would like them to have, like, that they havta- I dunno how much I'm gonna scaffold on the actual test, but otherway. A problem like the Kim, Allison, Lightning, Whirlwind whatever.

Pat: mhmm

Augusta: A problem like that. A problem like a time of- frame of reference problem. And then I was thinking that I could like give them a scenario and even like write the equation but then make them label what every aspect of the equation, like, represents.

Pat: mm

Augusta: That was actually- and I'll show it to you. The homework that I never gave 'cause I just didn't get to it?

Pat: Yeah.

Augusta: That's what it was.

Pat: oh. Label everything?

Augusta: Yeah, and I still want to do it ... ... [thunder] [thunder]crap!

Pat: That's close by.

Augusta: yeah

Pat: [blinds closing] My poor doggies are outside in the back yard

Augusta: ohh.... well, [Student] got her wish. I doubt they're running the mile right now. [blinds] damn... [blinds] there we [blinds]there we are. there we go. and so just some thing like

Pat: Yeah! this is good.

Augusta: That's what I wrote, and I never handed out. So something like that on the test.

Pat: Umm... this could be our homework.

Augusta: For tomorrow?
Pat: yeah.
Augusta: ok
Pat: uh.
Augusta: cause I eve did a frame of reference in there cause I knew that we were going to do Robin Banks next Of course I didn't hand it out, so they never saw it. I was trying to, you know, lead them up to it, but I didn't hand it out, so they never saw it.
Pat: ok. Yeah, that's good.
Augusta: ok. So do you think like that's cause that's really only based on one thing to, should I do like a back to it as well, maybe like one where I have an x minus the time. so I pretty much look off the other person's time of reference?
Pat: mm
Augusta: mmkay, so I'll do a back to it. Or a second sheet, that would- let's have front-back.
Pat: well let's see ....
Augusta: Or I could ask them to
Pat: well, here's something else you could do.
Augusta: We can write all over this ...
Pat: Well, here's what I was thinking. Underneath here. [Thunder] do the same thing, in columns.
Augusta: for the other point?
Pat: Say, d1 is uh- 75 times x minus five. is that right?
Augusta: yeee-es.
Pat: ok. umm d2 is uh 50 x. etcetera. It's the same questions. ..I mean, just have them side by side.
Augusta: mhmm
Pat: make them both about that scenario. .... [thunder] [thunder] ... ok and perhaps break this up. [thunder] Ok and then this also would- this would be over here.
Augusta: right right
Pat: alright? And then you would say.. colon.. ...ok x stands ..for ....uh x plus five stands for what. 50 [thunder]
Augusta: right
Pat: alright?
Augusta: ok, ok. so not even like, d1, but 50 x plus five.
Pat: yeah, cause that's where d1 gets it's meaning.
Augusta: right. mmkay
Pat: ok, and then, yeah- equation number three that- just leave that as it is. What does equation three represent? Equation four represent? And the same thing over here.
Augusta: ok
Pat: Just have two columns. You know how to do columns in Word?
Augusta: mhm!
Pat: Section, setting... alright, ok good.
Augusta: I'm ok with Word.
Pat: Alright.
Augusta: I'm good with Word. Ok, so that can be their homework tomorrow. We'll probably spend a huge chunk of the hour talking about whatever, and that's fine. Robin Banks. And then, ok. If I wanna test them Tuesday.
Pat: Ok, I'm gonna .. go put this in front of the
Augusta: Oh.
Pat: so that
Augusta: make sure it Cause you can see it?
Pat: huh? No, so that I mean I should have done this when we first started.
Augusta: So you wanna put it closer, or I dunno how good you can like.
Pat: I should have put it closer at the beginning. So that then whoever is watching the videotape would know what we're talking about.
Augusta: Right. ... ha. ...
Pat: Oh, and tomorrow,

Augusta: mhmm? oh yeah, set it up.
Pat: Just set it up. It's just got a little snap thing that you put in. Get it set up so that ...it pretty much captures the very front end of the room.
Augusta: ok
Pat: Um, and then, just press the red button on the top. See how it's blinking?
Augusta: mhmm
Pat: That tells you it's recording.
Augusta: ok..ok.
Pat: alright? [Thunder] ... Another thing, that you might do is you say, ok time-out with Willie and Robin. Don't try and recover.
Augusta: ok.
Pat: Time out. [Points at homework worksheet] Let's do this. ...
Augusta: ok ..or maybe they could try to redo the problem [inaudible]
Pat: I mean, that's another option. ... your call. But that's an option also.
Augusta: mmm ...
Pat: Cause this is a nice- It's a scaffolding activity, but it's not scaffolding in the same way, it's not scaffolding to an answer.
Augusta: right.
Pat: It's scaffolding like, here's a new issue.
Augusta: mhmm ... I wrote that all by myself! Ha ha ha ha ha ...
Pat: ... that's-

08-30 Review for Test

Augusta: So, you know, to be honest guys? I just wanted today to chance for you to ask me questions, and ta- figure out any ...problems you feel like you're having. so as you guys have been looking through this Do you guys wanna tell me anything that for whatever reason continues to confuse you? Continues to confuse you? ... ...I'm gonna keep track of them, then I can talk about them. Nothing? Yes dear?
Student: I don't have the Robin Banks one.
Augusta: You don't even have the Robin Banks one. Ok it's- If ever you are absent, first of all, check the webpage. 'Cause it is up there. But second of all I always have extras, in this tray.
Student: I turned it in yesterday.
Augusta: Oh, you turned it in yesterday. And you want it back. Where did I put it? Kay. See me at the end of class. I'll find it for ya. Ok? Well, ok, How about this guys? Something that I think we were having as misconceptions, how about that? You don't feel like asking me.
        Is first of all, when you guys has an expression Where, somebody was traveling ..and I'm making this up. Like 15... miles per hour. And they left two... hours.. after somebody else. Remeber these? Where there was that time gap? Kay? What I feel like a lot of us were forgetting Were the parentheses? Kay, raise your hand if you forgot parentheses at least once in your life. Kay. That's cool.
        Um. ..Why do we need the parentheses? [Student]. Why is it so important that I use parentheses in this problem.
Student: So you do that portion first?
Augusta: Ok, so that I do this portion first. And why is it so important that this portion gets grouped?
Student: Umm... because ..x would be the time of the other person that left?
Augusta: X is some unit of time, good.
Student: And the minus two is the hours after. wait. er. like. Hours after.
Augusta: Right. They left two hours after. So it's another unit of time, right?
Student: Yes
Augusta: You gotta group your units of time. OK. Think about this with me. If you just had- cause a lot of people have like 15 x, and then, you know. minus two. ok. raise your hand if you've written it like this at least once in your life. Kay, we're-you know- we're all there.

So think about this. If you take the rate at which someone is traveling and you multiply it by how long they've been traveling. What does this answer represent? If you take a speed and multiply it by how long they've been on the road, what does it represent? [Student]

Student: How far the other person's gone?

Augusta: It's a distance. Kay. we don't even really have too much of a context in this, but it's a "how far" it's a distance. yes? ..Now. I made this, but you know, we left two hours later. So this whole minus two, what does it represent, [Student]?

Student: Umm..taking away

Augusta: Taking away what?

Student: like..... like x?

Augusta: Well, nope. I made up my little scenario. So what does my two represent?

Student: two hours?

Augusta: It's a number of hours. Kay? So think about this. If you don't put parentheses to group Your units of time. Then you're taking a distance and subtract a number of hours. Does that make sense? Can you 25 feet and say 25 feet minus one hour?

Student: Hm!

Augusta: [Student]'s already laughing at me. Does that make sense? No, you can't take 25 feet and subtract an hour from feet. Kay, it makes no sense.

So as you guys are setting up your expressions on your assignments and on your test tomorrow. That's one of the main reasons that we have to ask ourselves "What am I representing?" If you just think about it. If you write an expression like this. Then you've just written an expression that takes time away from a distance. That makes no sense. Kay? You can take time away from another unit of time. And that's ok. right? So that's yet another reason why you guys need to be thinking about what everything represents. mkay?

Cause as you guys are going along, and you write something bogus like this if you even just take two minutes! Two seconds! Ta ask yourself "What did I just write?" "What does this supposedly represent?" And you ask yourself "Ok, what does this part represent?" You guys are smart young people, so you'll know that it's distance.

And then you ask yourself, "What does that represent?" And you'll look back at the scenario and like, "Oh, that's my two hours." So you say, ok "[Student], self, I'm taking two hours away from a distance." And then self would say, "Dude. No. What're you doing?" right? So you guys need to think about these things. kay?

You need to be thinking about these things.

11-07 Intervals with one student.
5<x+2<10

Augusta: It's that your interval when increased by two is between five and ten.

Student: So it should be seven- it should be like two...plus them two more on each side?

Augusta: Well, maybe. Think about it. think about values that would work here. So like imagine if you just drew it kind of blindly, you'd have the five and the ten, right?

Student: mhm

Augusta: Cause you didn't think any better, so there's your five, there's your 10.

Student: yeah.

Augusta: So that would mean your interval starts at five and goes all the way up to 10. Right? So if you start at five, then let's plug five in. Plug a five in there.

Student: kay?

Augusta: And it would do what?

Student: add two?

Augusta: And become a..?

Student: Seven.

Augusta: Seven. So, if you start at five Then you actually didn't start at five. You started at seven. ..because your interval when increased by two! has to be between five and 10. So starting at

five actually isn't starting at five. It's starting at seven.
Student: ok.
Augusta: So where do you need to start so that when increased by two! it starts from five to ten?
Student: ...Three?
Augusta: So, maybe. let's check it. If we started at three go ahead and plug three in.
Student: ok.
Augusta: Three, increased by two.
Student: Is five.
Augusta: Starts at five.
Student: ok.
Augusta: so maybe you should start at three then. Good. Do the same thing. Think the same way for the other side, too. Do you really wanna stop at 10? I dunno. Plug ten in, and see if it fits. It might not.
Student: ok.
Augusta: So think about it. Good.

11-09 Student presenting a homework solution
-4≤2x≤10

Augusta: So there's the original problem negative four is less than or equal to two x is less than or equal to ten. Tell us what you did.
Student: Umm... I divided by two on everything.
Augusta: kay.
Student: And then I got negative two is less than or equal to x is less than or equal to five ... and the, I did filled in circle on negative two and then a filled in circle on five? And I connected them?
Augusta: Ok, so again. I mean, she divided everything by two because the x is being multiplied and that's fine. But think about it. The interval you need to show me isn't really between -4 and 10. The interval that you- that you're showing me When It's Doubled. When it's doubled is in between -4 and 10. So you're actually between -2 and 5. Cause when you double -2 and 5 you're at -4 and 10. That's exactly where you needed to be. And so, she she shaded in between but we should probably check, just to make sure.