Function Transformations

A function is something you "do" to a number

- $f(x)=2x-3$
- I have a number, x
- I "do" f to it. (double, subtract 3)
- I get $f(x)$ out.
- This pattern works no matter what I call my x.

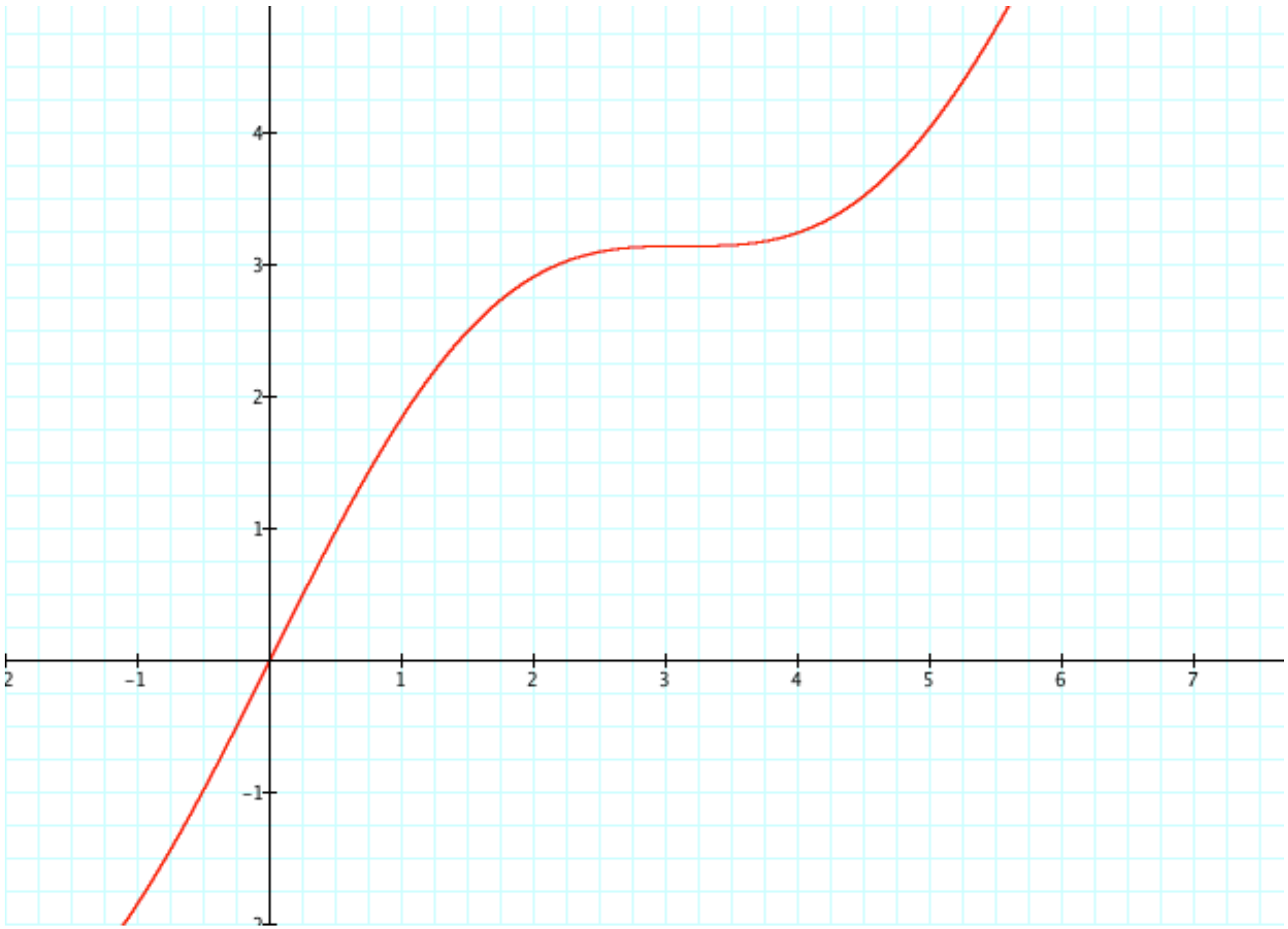
Arrow Notation

- x --f--> f(x) [I take x, I do f to it, I get f(x)]
- 2 --f--> f(2)
- p --f--> f(p)
- 3a-7 --f--> f(3a-7)
- g(x) ---f--> f(g(x))

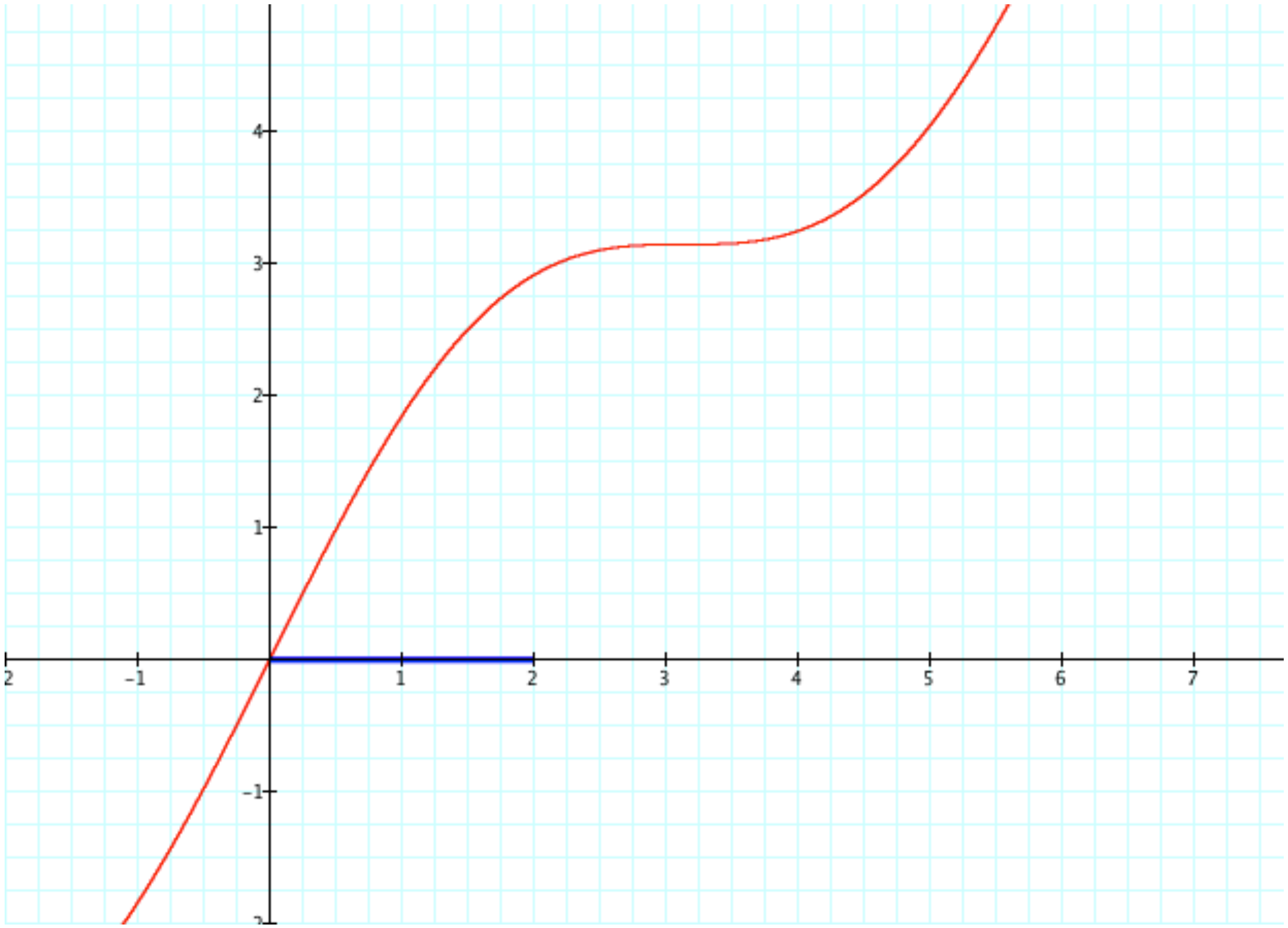
What does it mean to "do" a function on a graph?

- x is a horizontal length
- $f(x)$ is a vertical length.
- f is the curvy thing that tells you how to get from x to $f(x)$.
- you "do f" by starting at the end of x, and going up and over to find the end of $f(x)$.

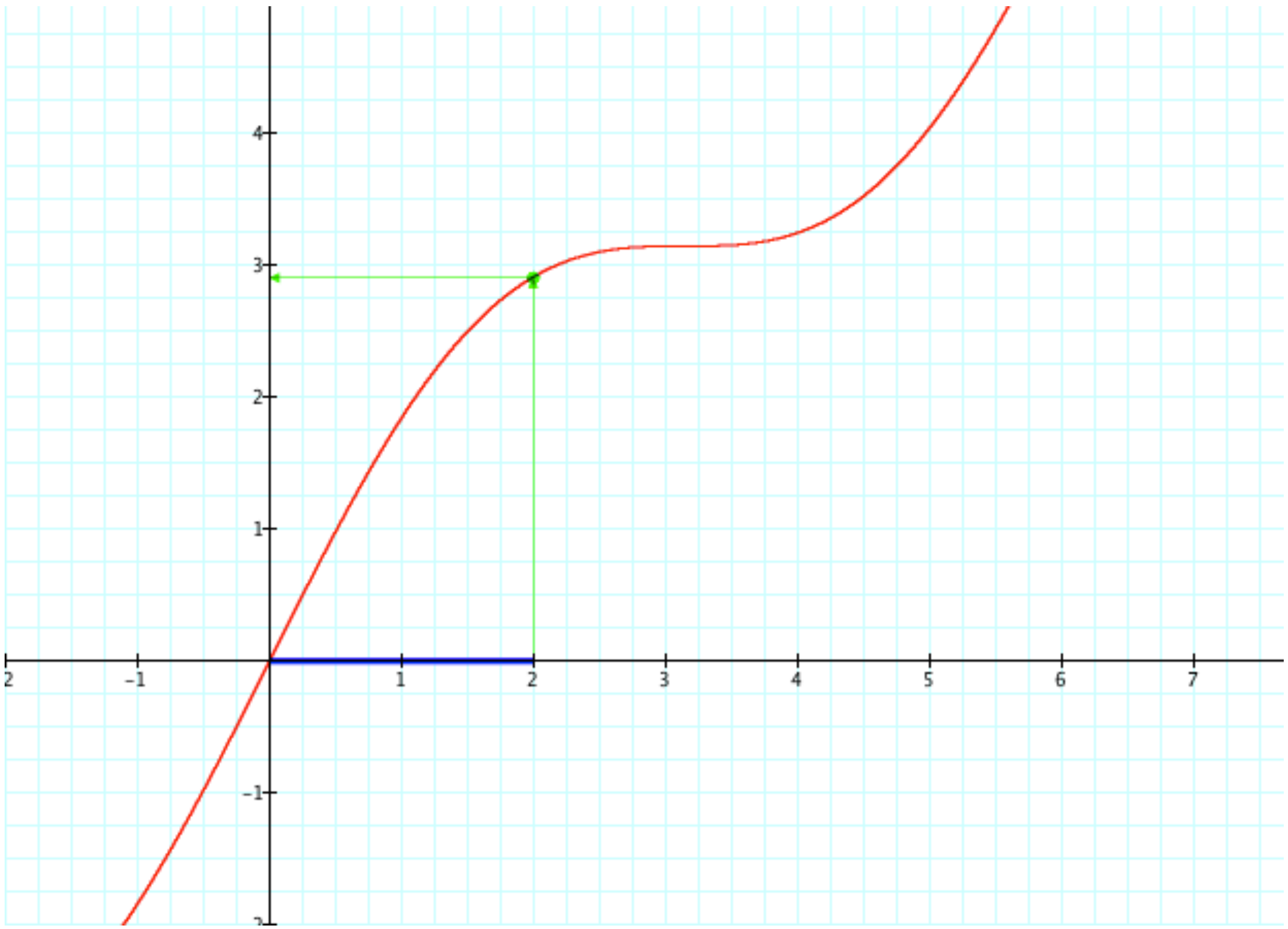
this is f



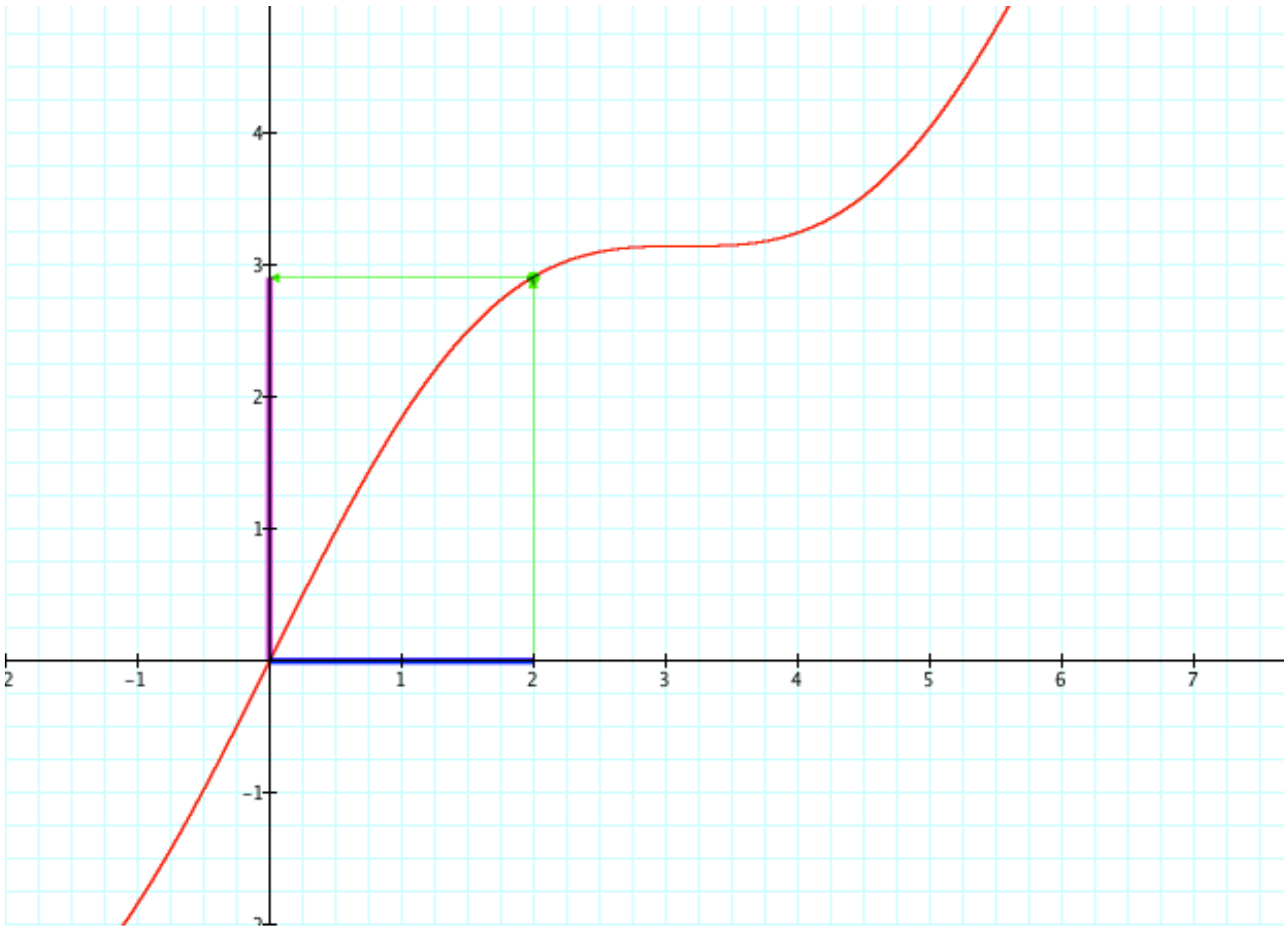
this is x



this is “doing” f to x



the result is f(x)



Adding functions

$$
\begin{tikzcd}
x \arrow[r, "f"] & f(x) \arrow[dr] & & \\
& & + \arrow[r] & f(x)+g(x) \\
x \arrow[r, "g"] & g(x) \arrow[ur] & &
\end{tikzcd}
$$

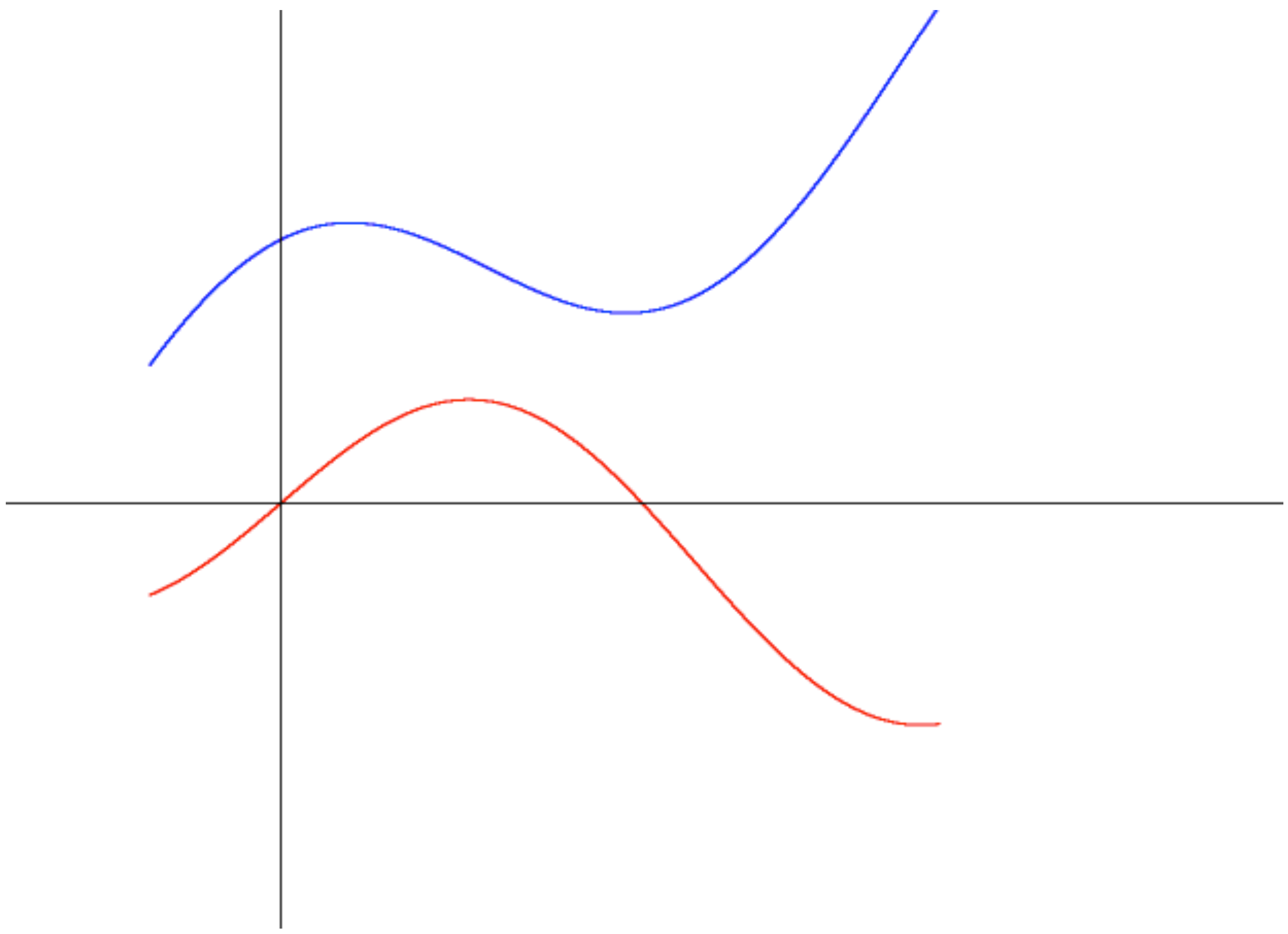
Adding functions

$$
\begin{tikzcd}
 & f(x) \arrow[dr] & & \\
x \arrow[ur, "f"] \arrow[dr, "g"'] & & + \arrow[r] & f(x)+g(x) \\
 & g(x) \arrow[ur] & &
\end{tikzcd}
$$

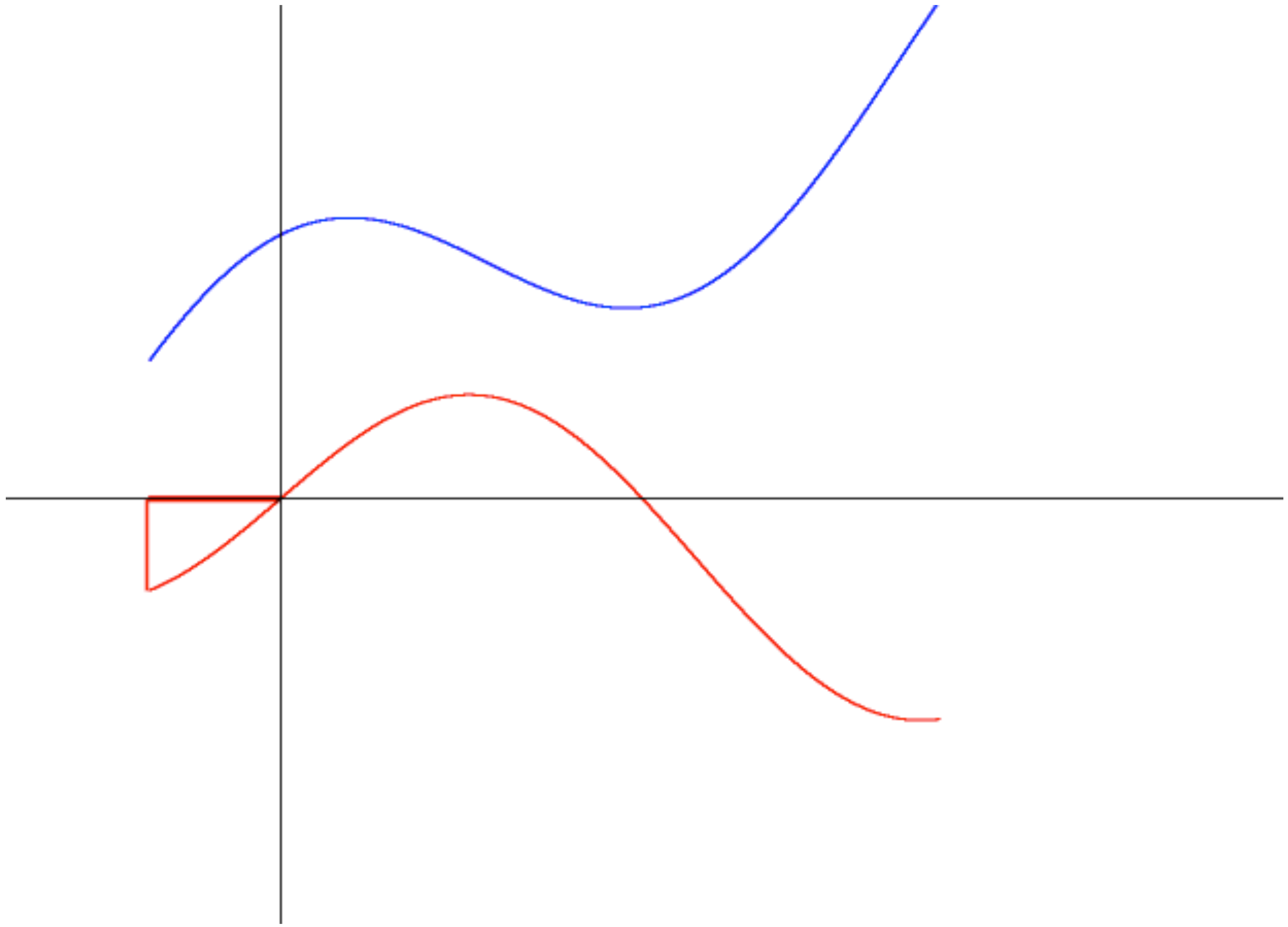
Adding functions

$$x \xrightarrow{f+g} f(x)+g(x)$$

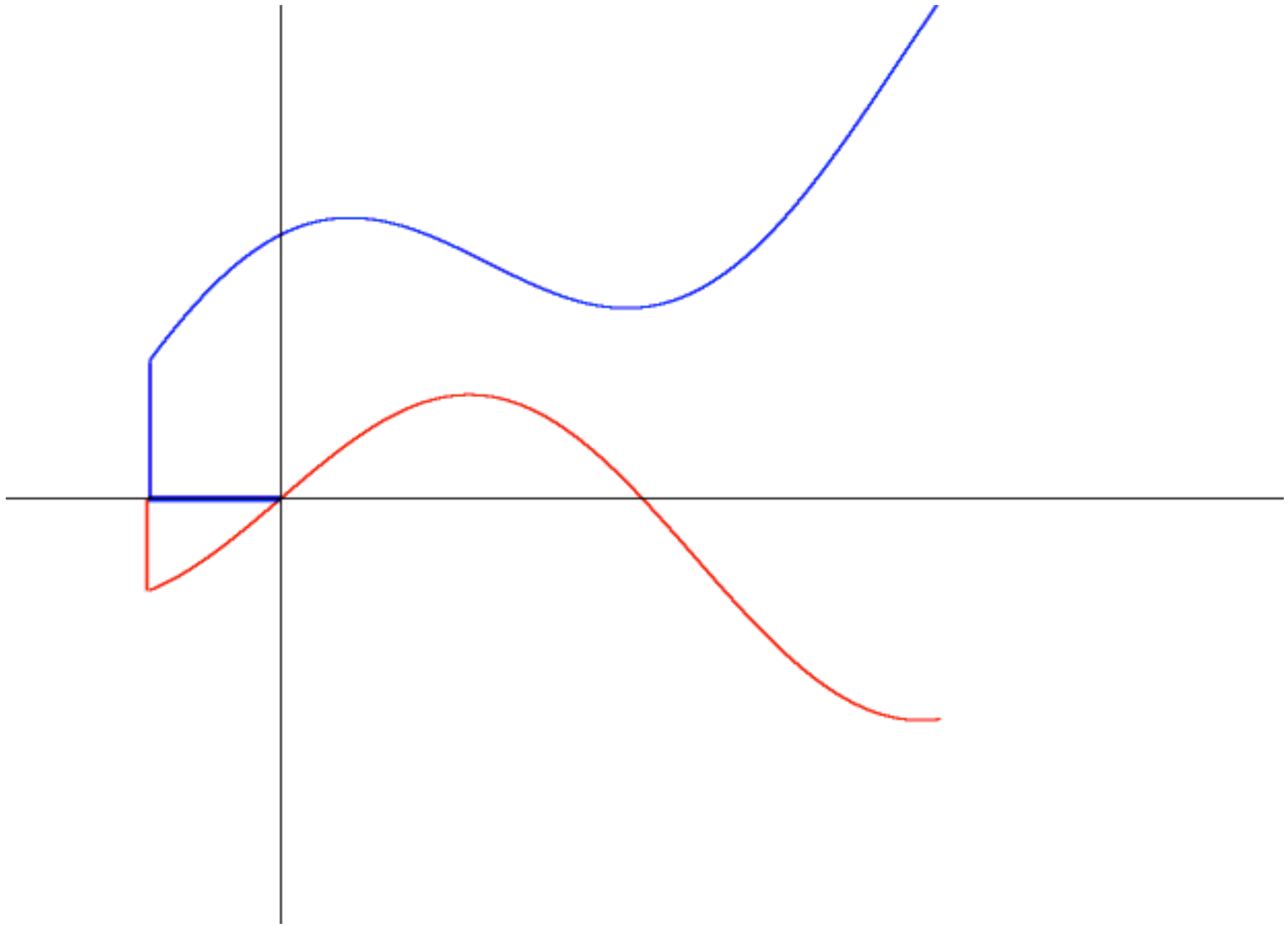
Adding f+g



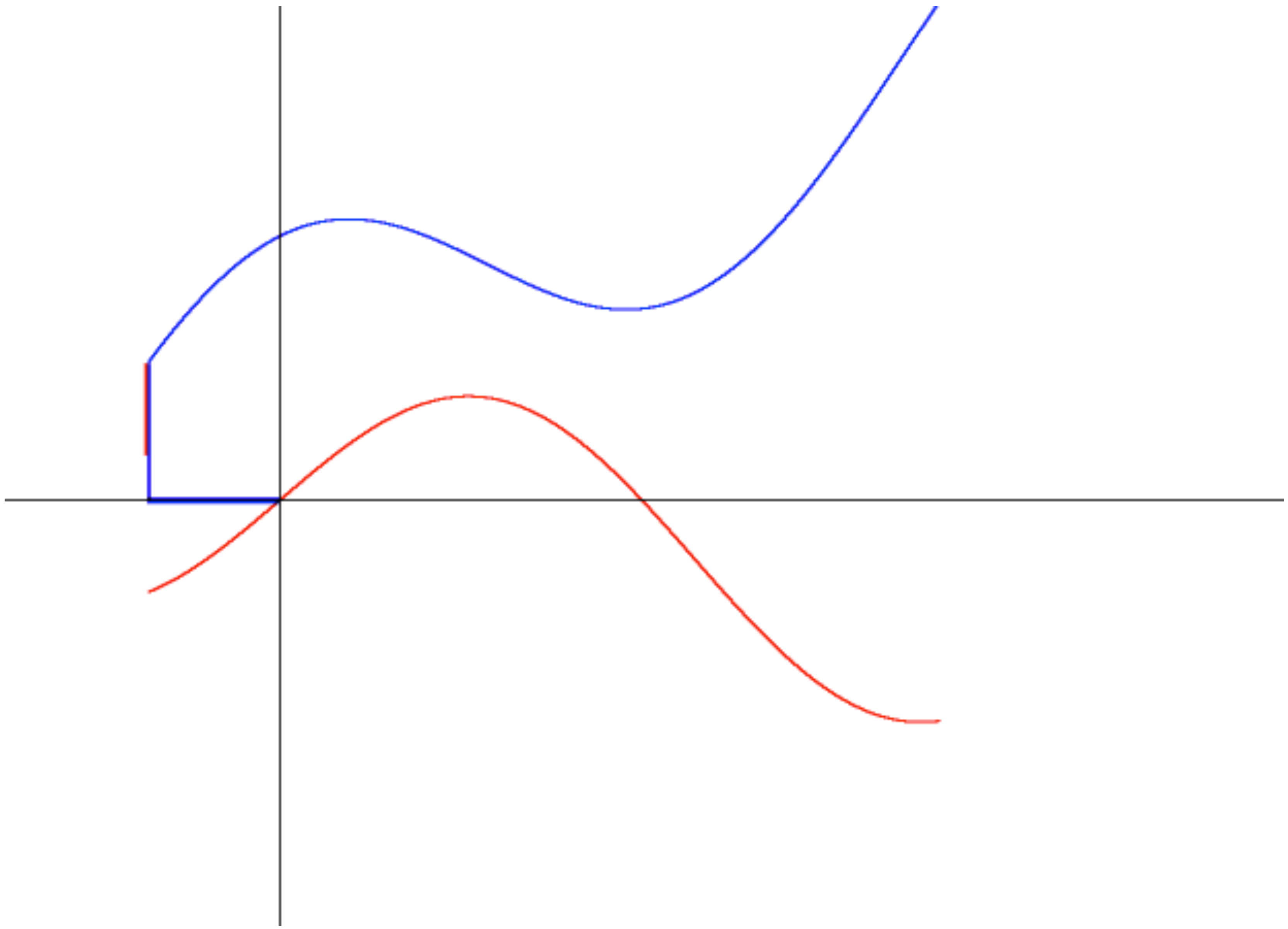
find $f(x)$



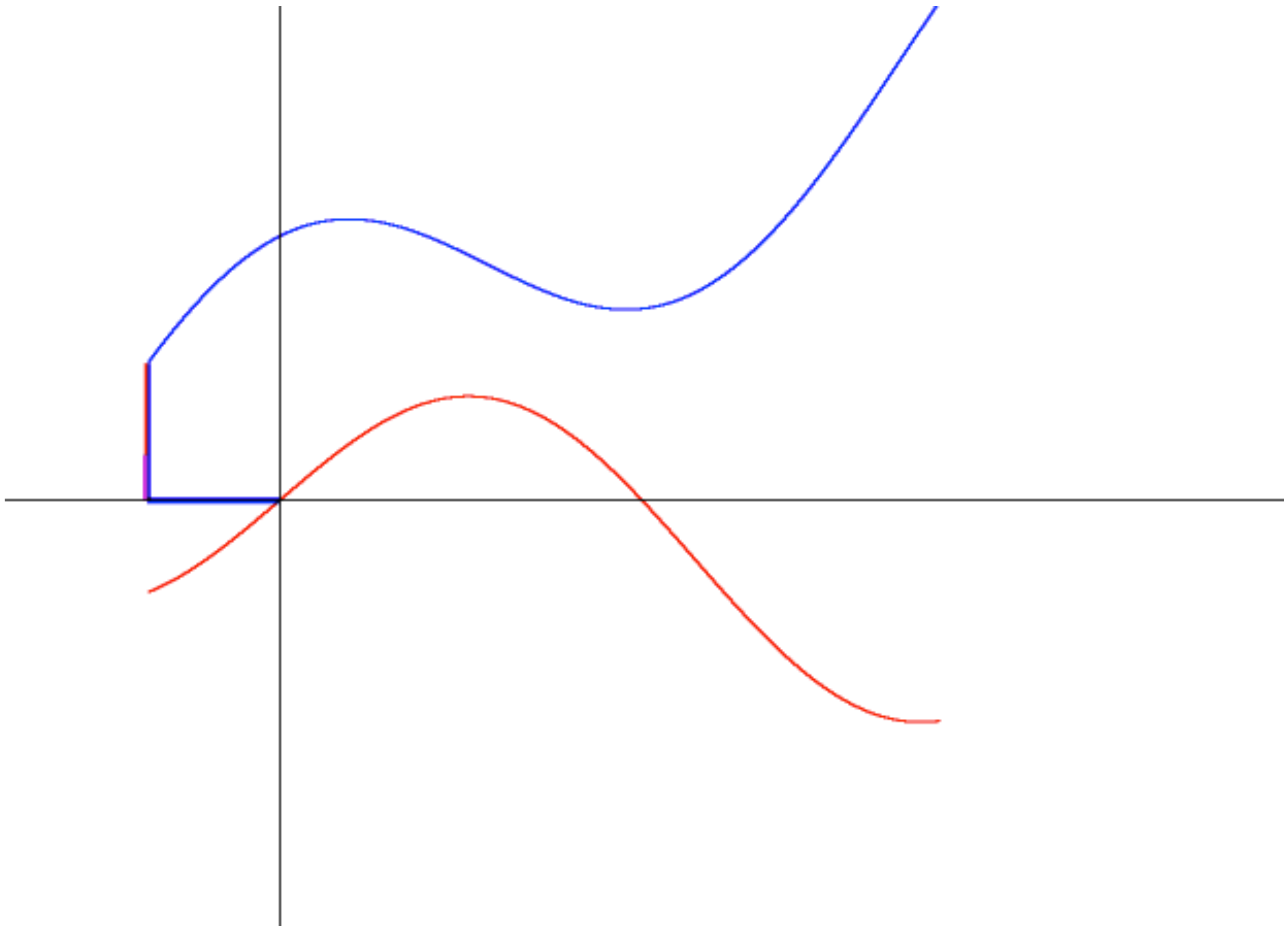
Find g(x)



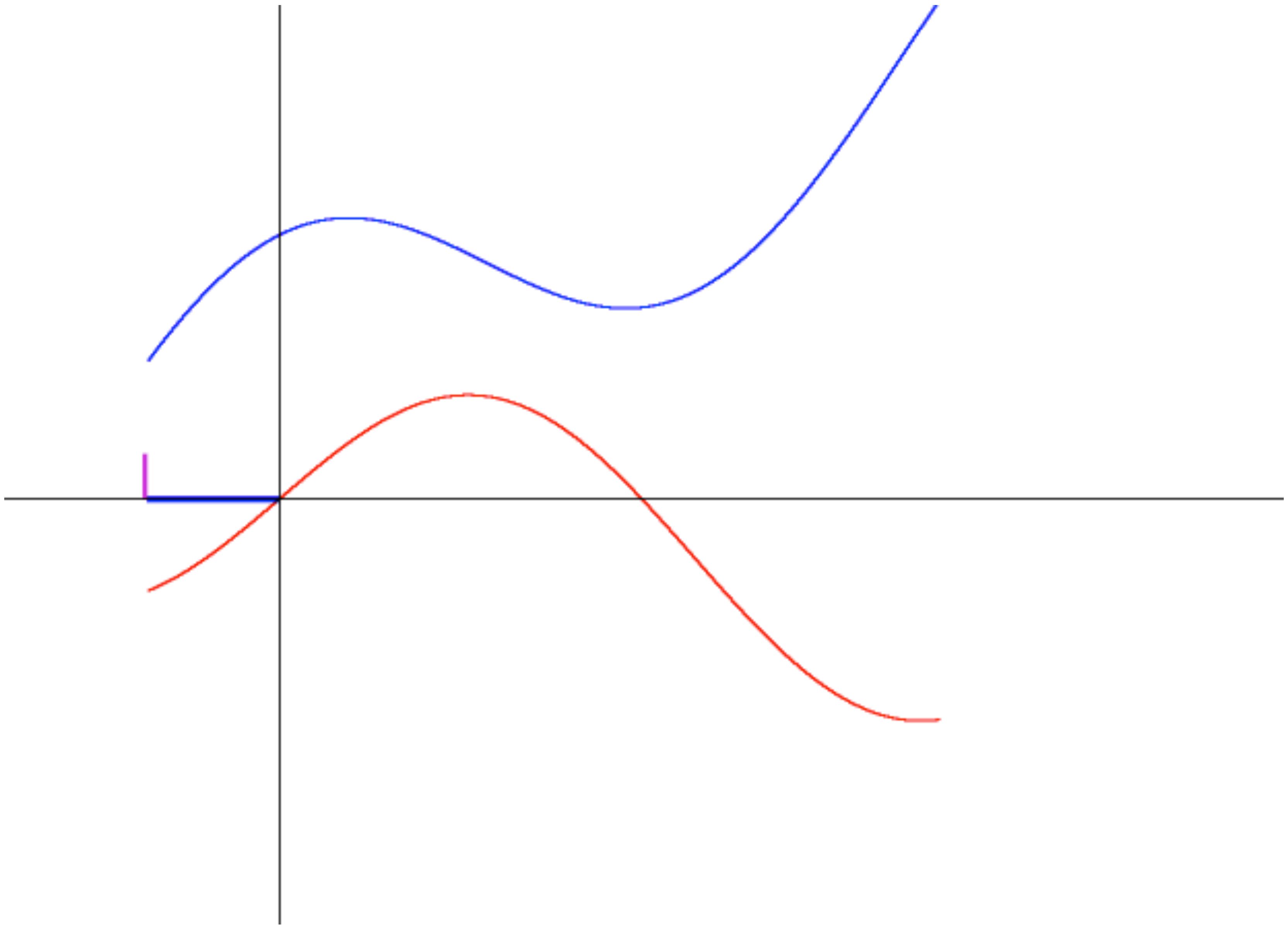
Add $f(x)+g(x)$



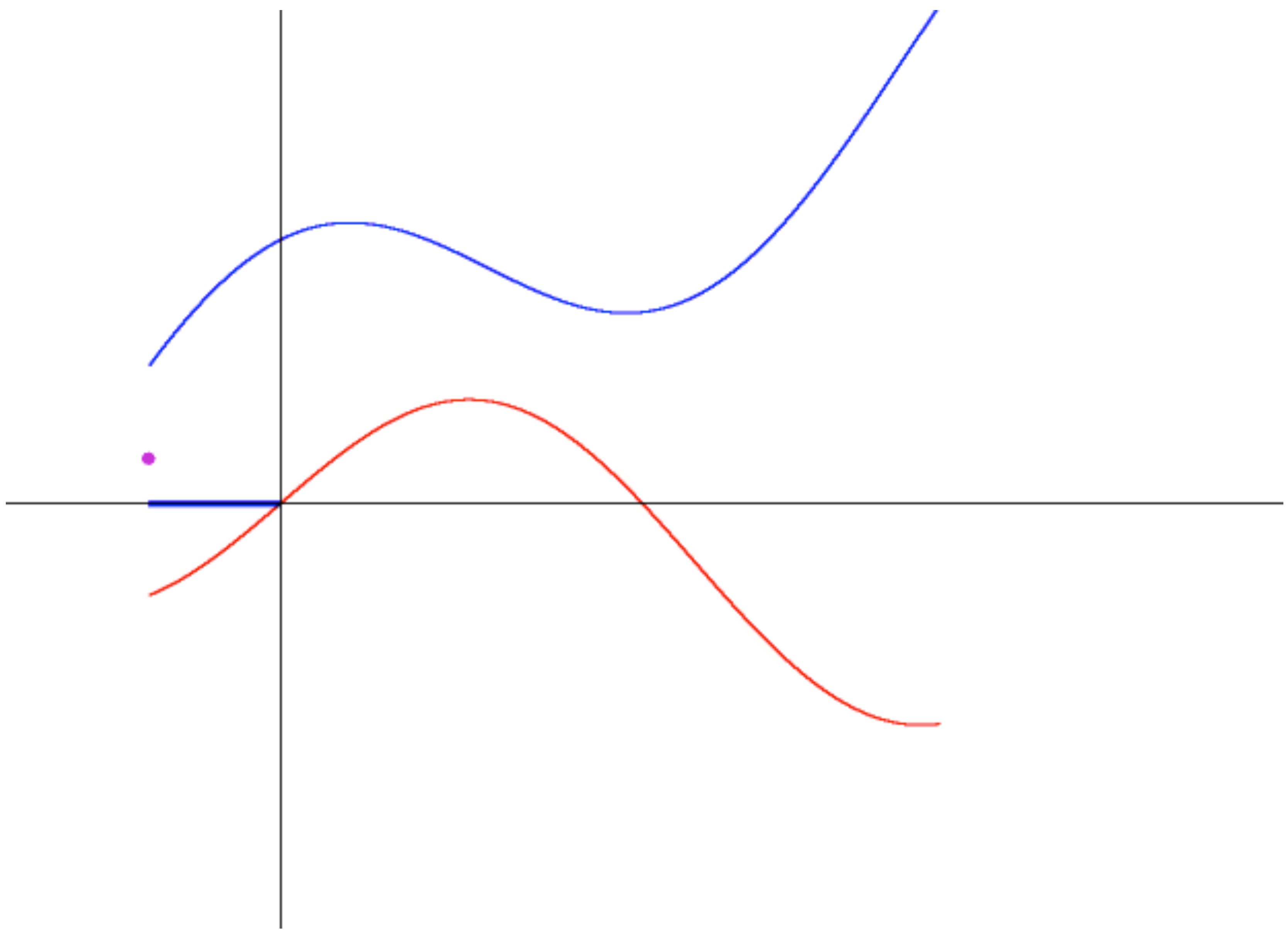
find $f(x)+g(x)$



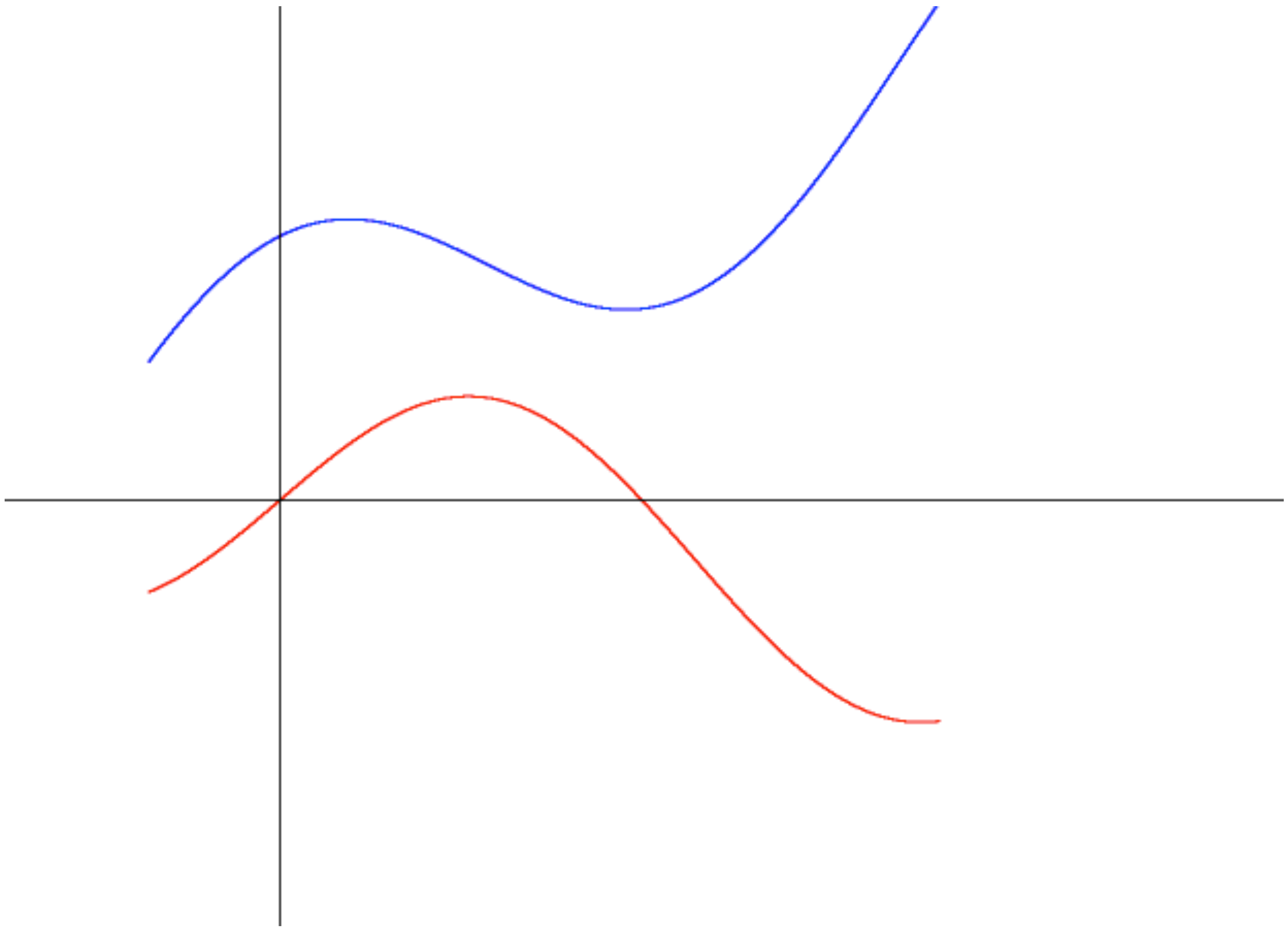
find $f(x)+g(x)$



plot f+g at x

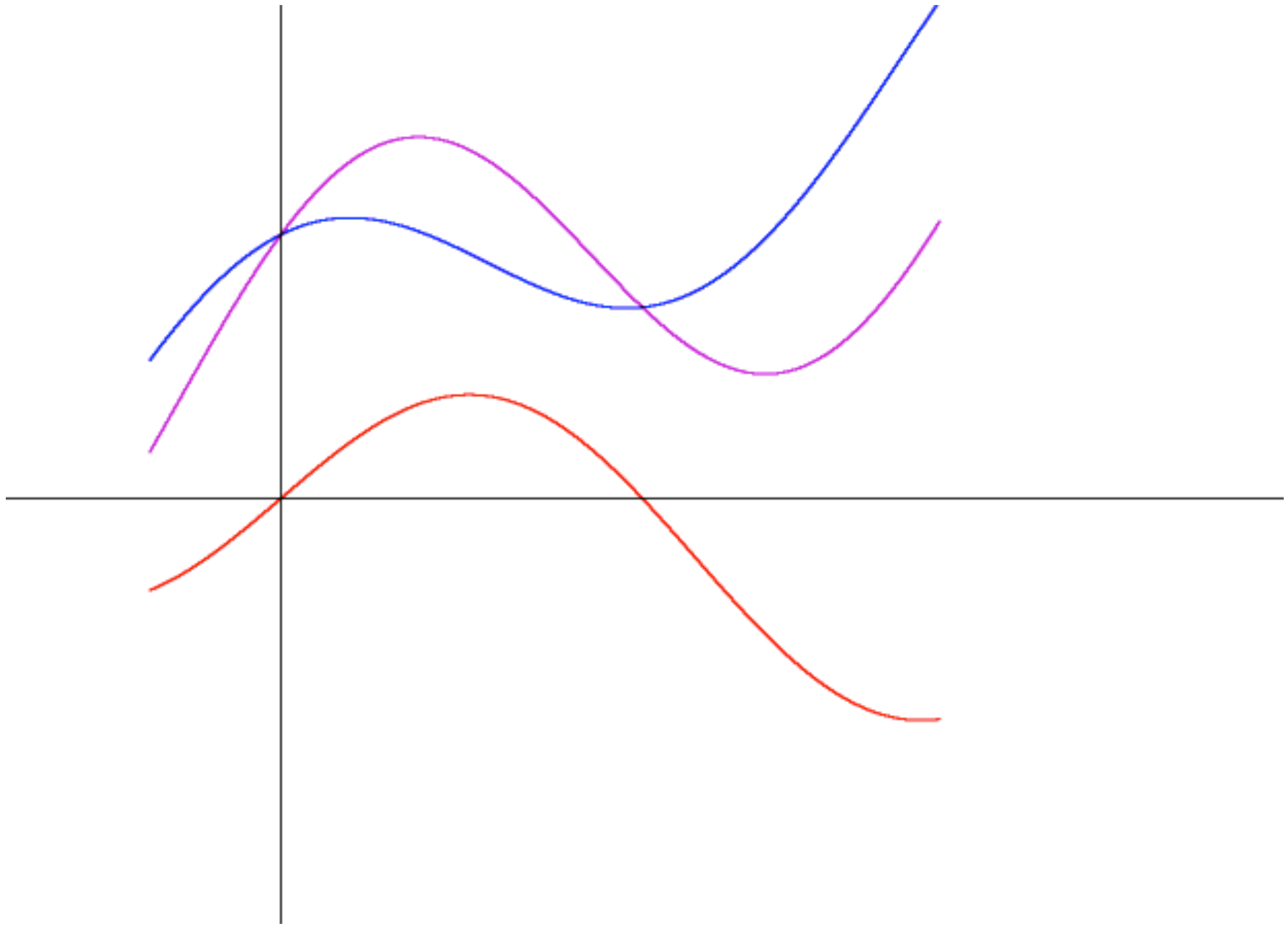


Your turn



Create a graph of f+g, You can use a straight edge, but not a ruler.

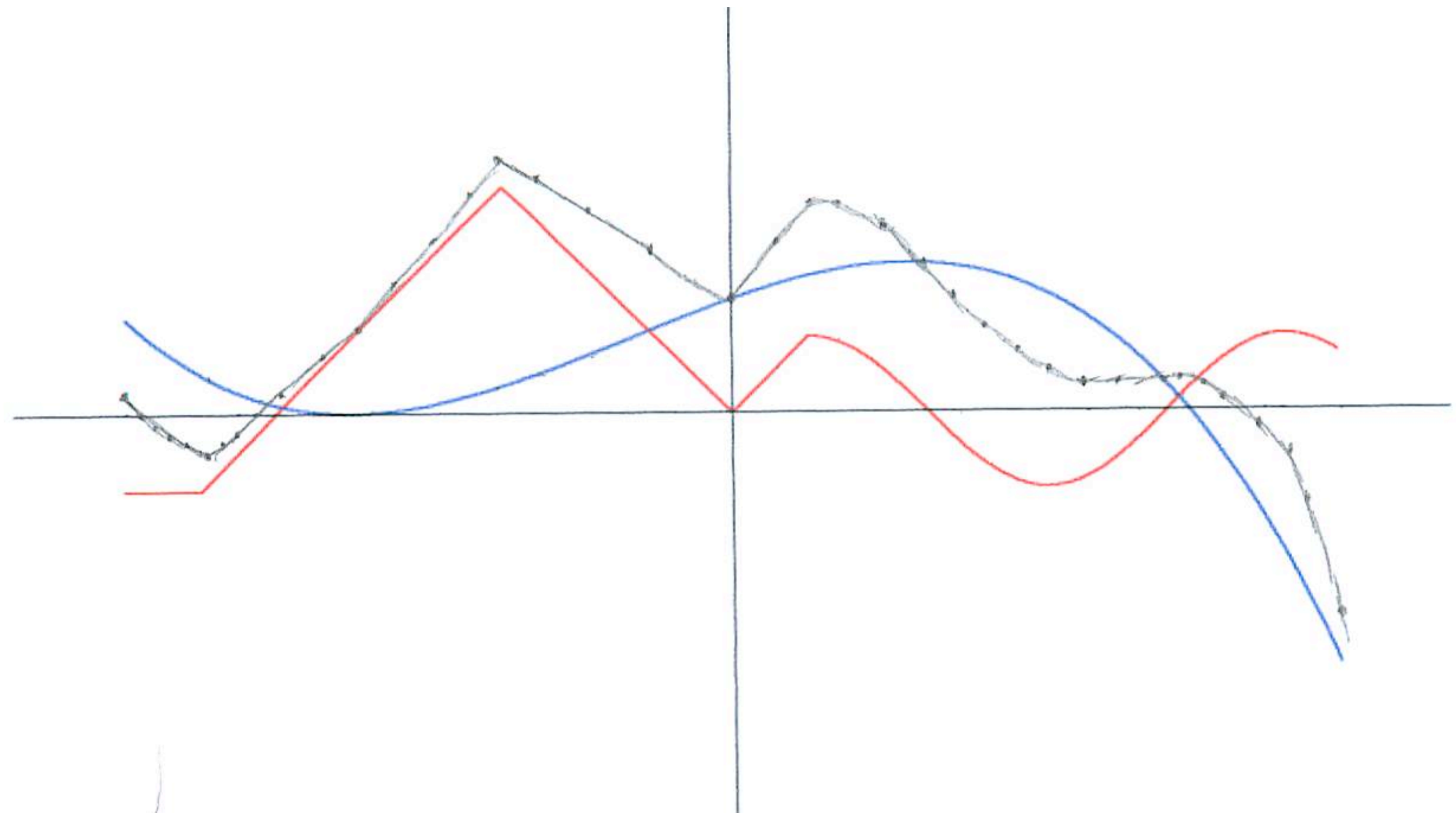
Solution



Why straight edge but not a ruler?

- So that students are adding **lengths** instead of **numbers**

With students



Composition

Composition

$$x \xrightarrow{f} f(x)$$

$$x \xrightarrow{g} g(x)$$

Composition

$$x \xrightarrow{f} f(x)$$

$$f(x) \xrightarrow{g} g(f(x))$$

Composition

$$x \xrightarrow{f} f(x) \xrightarrow{g} g(f(x))$$

Composition

$$x \xrightarrow{f \circ g} g(f(x))$$

Composition

$$x \xrightarrow{f} f(x)$$

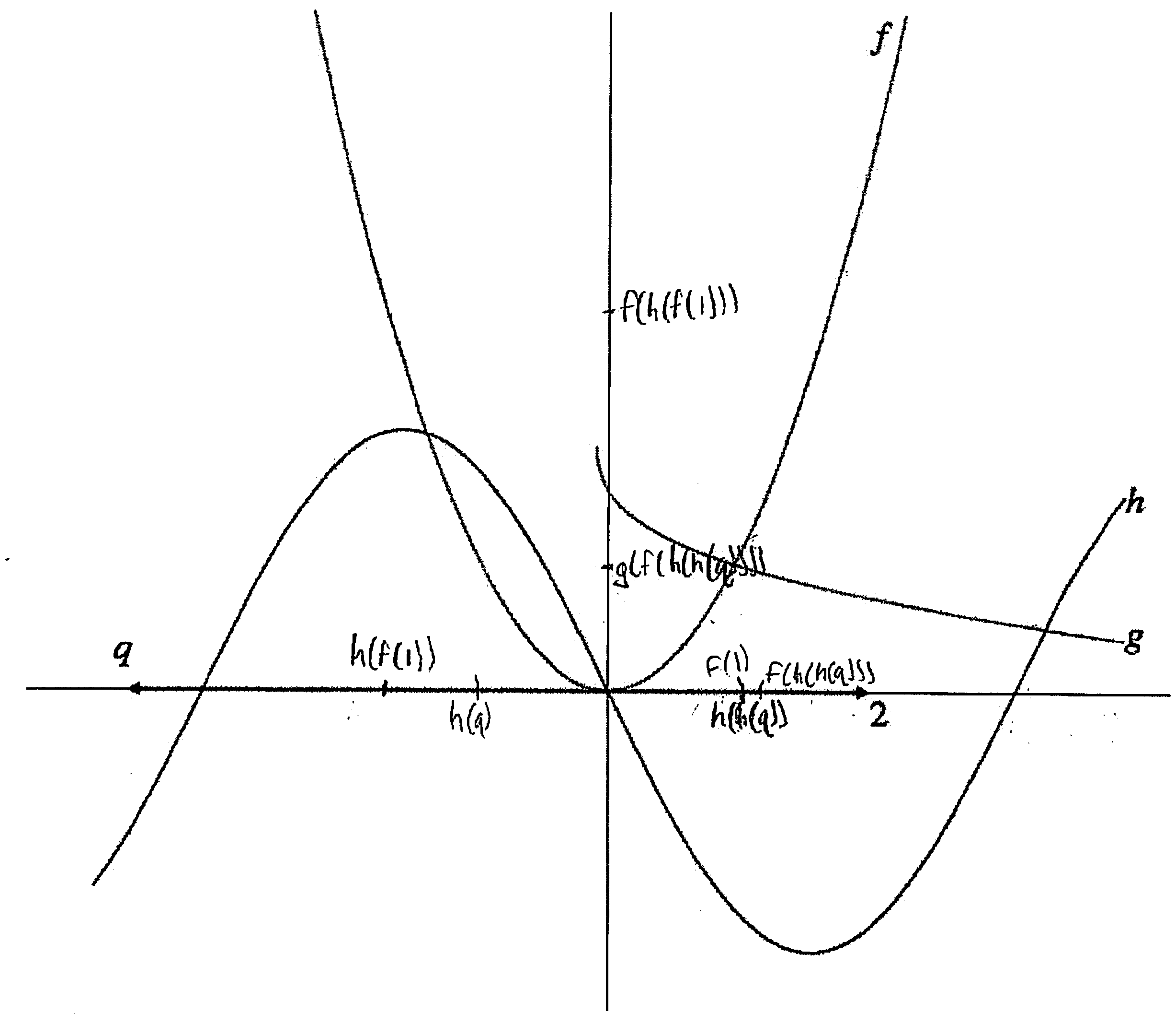
$$f(x) \xrightarrow{g} g(f(x))$$

This renaming step is **critical**.
When composing graphs
It helps students if they
rename the axes to mimic this

Composition

- Composition is harder, because it's directional. (it matters if f is increasing or decreasing)
- You can do it with points, but students tend to space the points too far apart.
- It's harder (but better) to force them to do it with intervals

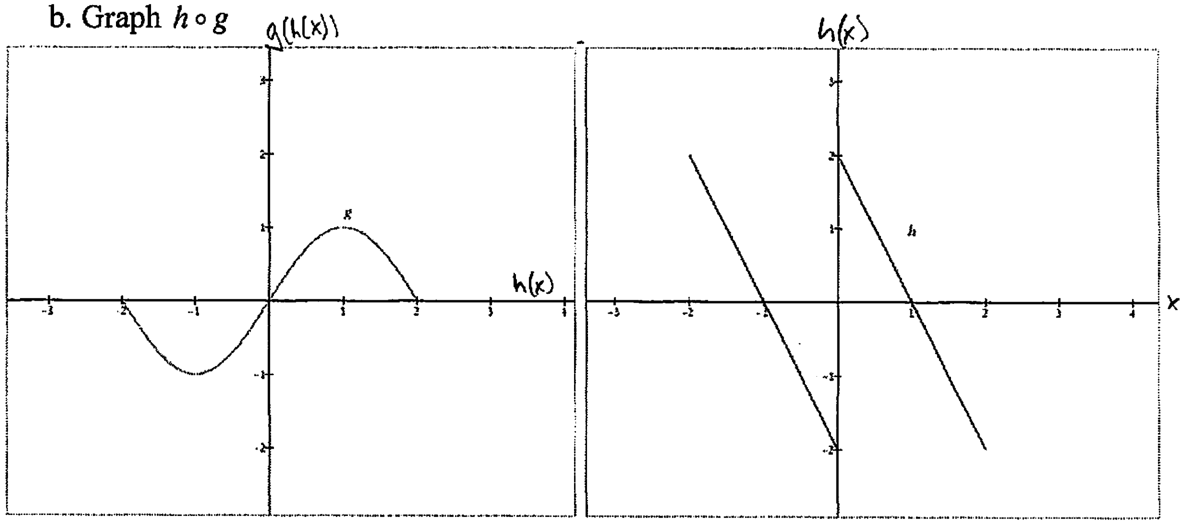
Composition with points

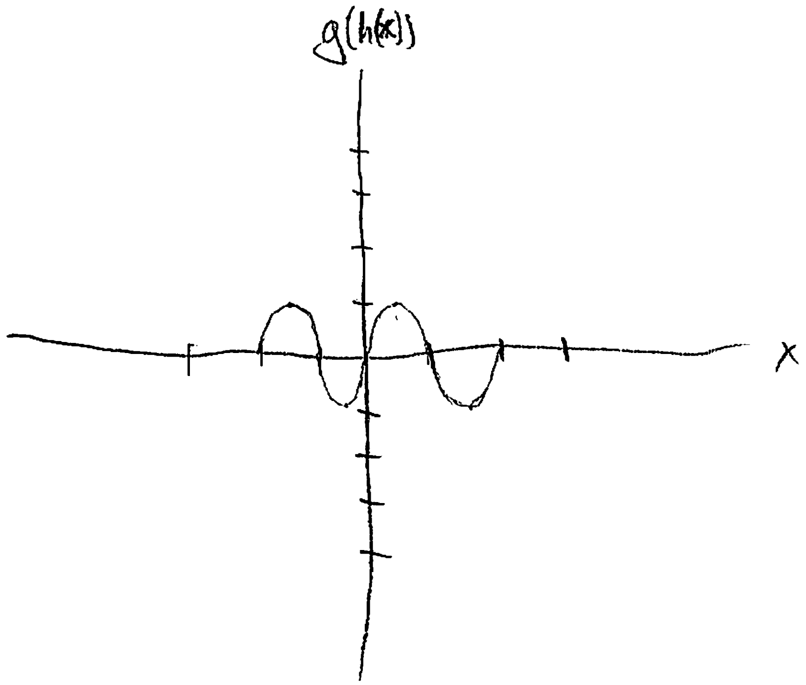


2. Same graph, fresh copy
   a) g(h(2)) not possible
   b) g(f(h(h(q))))
   c) f(h(f(1)))

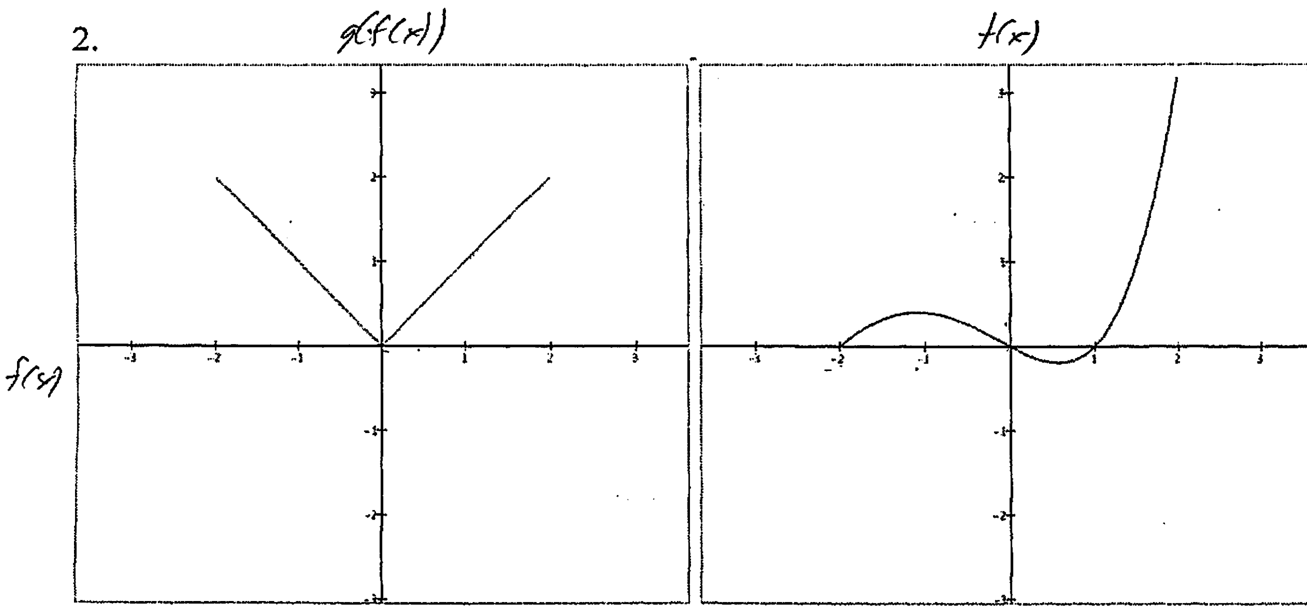
Composition with Axes labeling

2. a. Label all axes.
b. Graph $h \circ g$





Composition with intervals

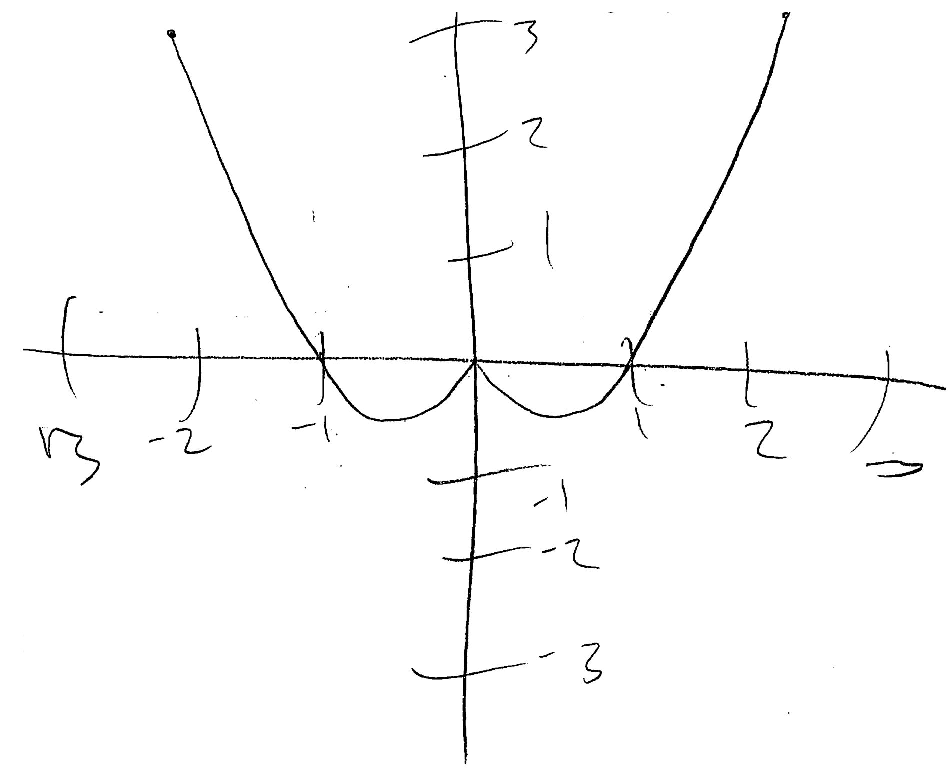


x: -2 -> -1
f(x): 0 -> 1/2
g(f(x)): 0 -> 1/2

x -1 - 0
f(x) -> 1/2 to 0

x : 1 to 2
f(x): 1 to 2

f(x) : 1 -> 2
G(F(x)): 0 to 3ish



What is this used for?

- Building transformations of functions out of parts.
- $2f(3x-7)+2$
- $g(x)=3x-7$
- $h(x)=2$ <--- always a shift up, because you're always adding the same length.
- $f(g(x))+f(g(x))+h(x)$
- What happens if you subtract 2 inside? $f(x-2)$. Use composition.

What is this used for?

- Building polynomials.
- $y=x^3-3x^2-7$
- $f(x)=x^3$
- $g(x)=x^2$
- $h(x)=-7$
- graph of polynomial is graph of $f(x)+g(x)+g(x)+g(x)+h(x)$