Exponential Growth

Constant Growth

Baka bank uses a constant growth rate policy for their SI8 investment accounts. The value of an SI8 account grows at a rate of eight dollars per year.

You have 100 dollars to invest.
What's the best strategy for investing it?

Simple Interest

Jodan bank uses a simple interest policy for their EZ8 investment accounts. The value of an EZ8 account grows at a rate of eight percent of the initial investment per year.

Patricia begins with $500 in her account

Simple Interest

Jodan bank uses a simple interest policy for their EZ8 investment accounts. The value of an EZ8 account grows at a rate of eight percent of the initial investment per year.

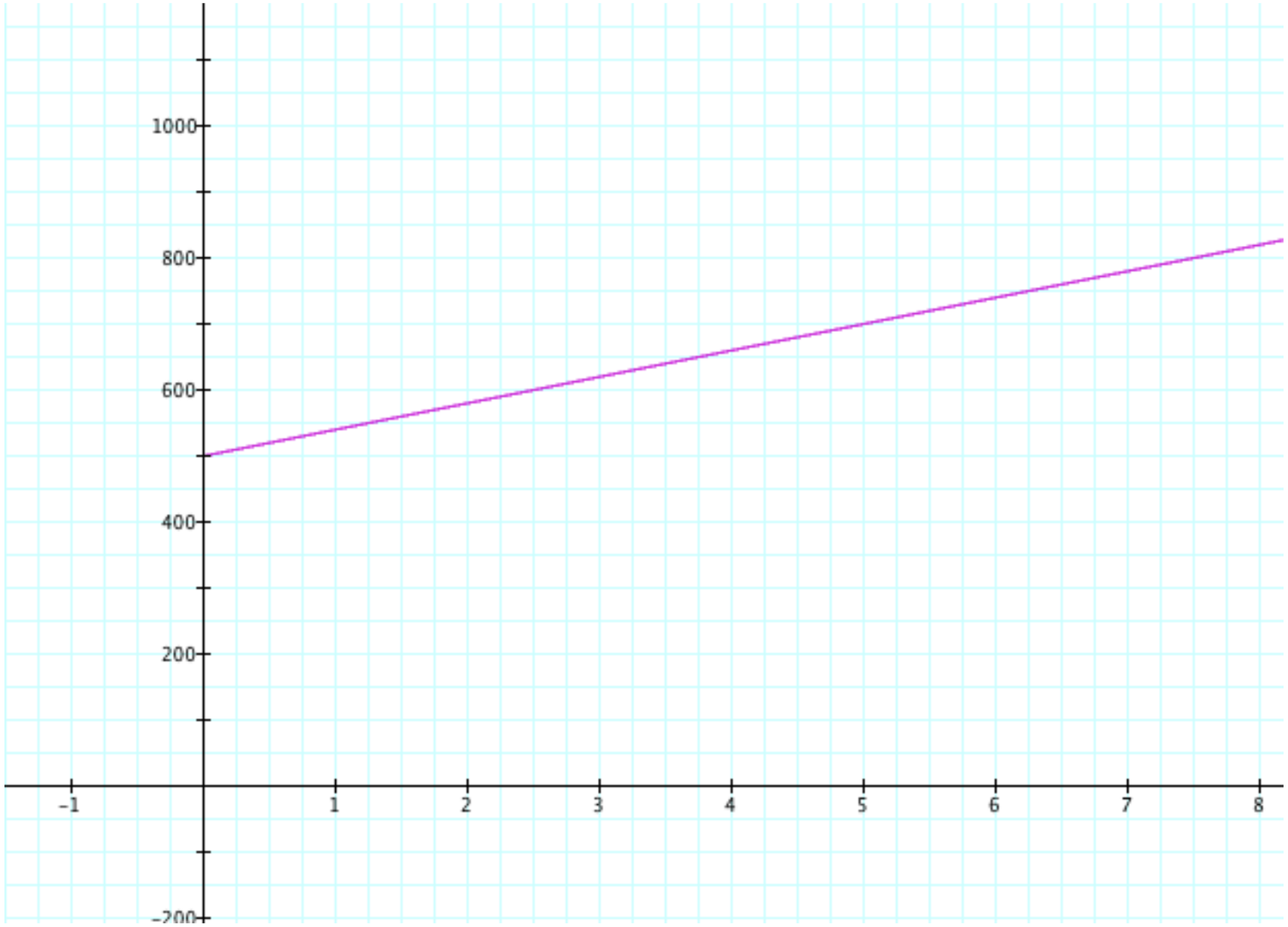
Patricia begins with $500 in her account

Patricia's account grows at a rate of 8% of $500 per year.

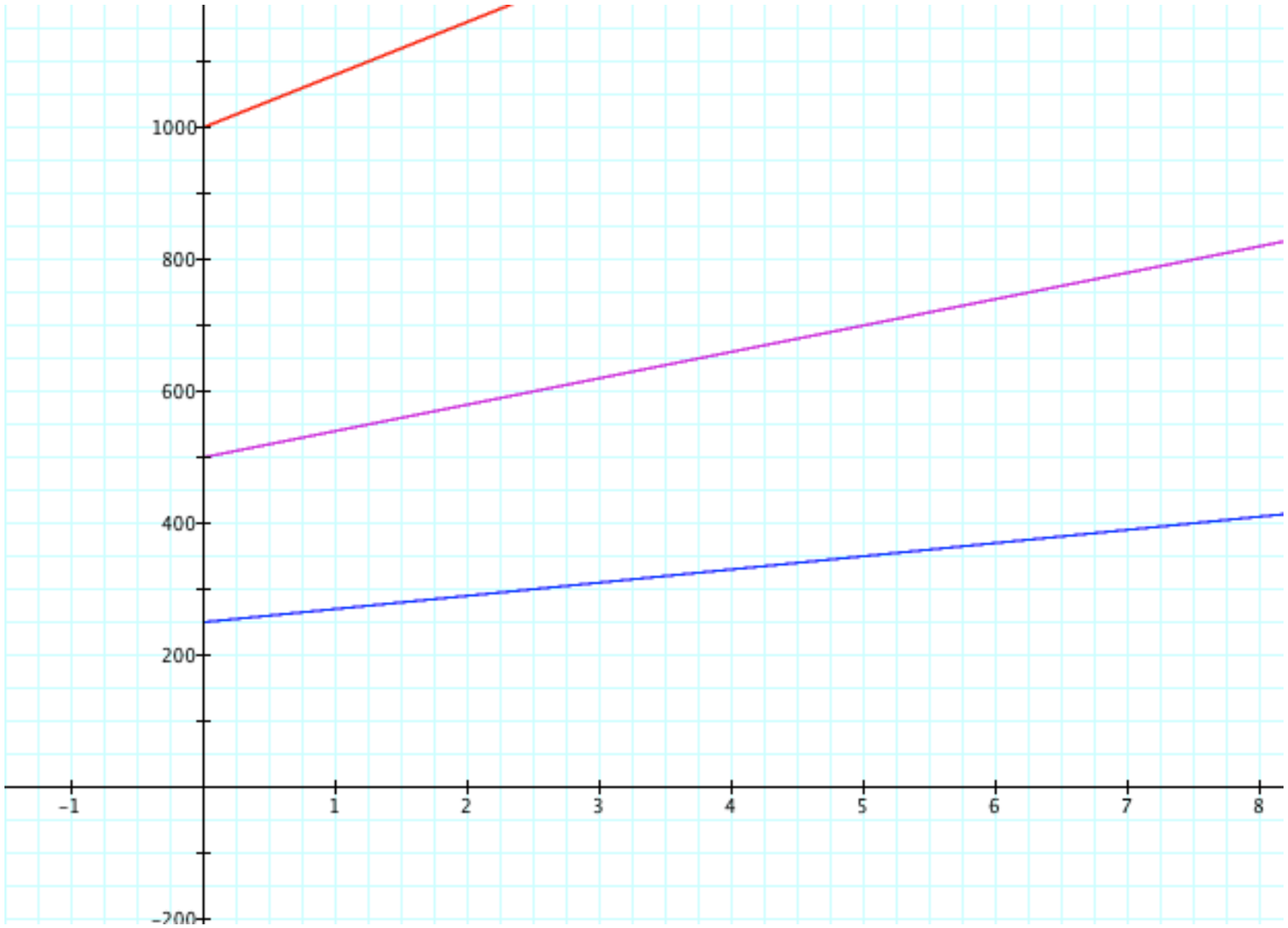
Patricia's account grows at a rate of $40 per year.

P=40t+500

Simple Interest



Simple Interest

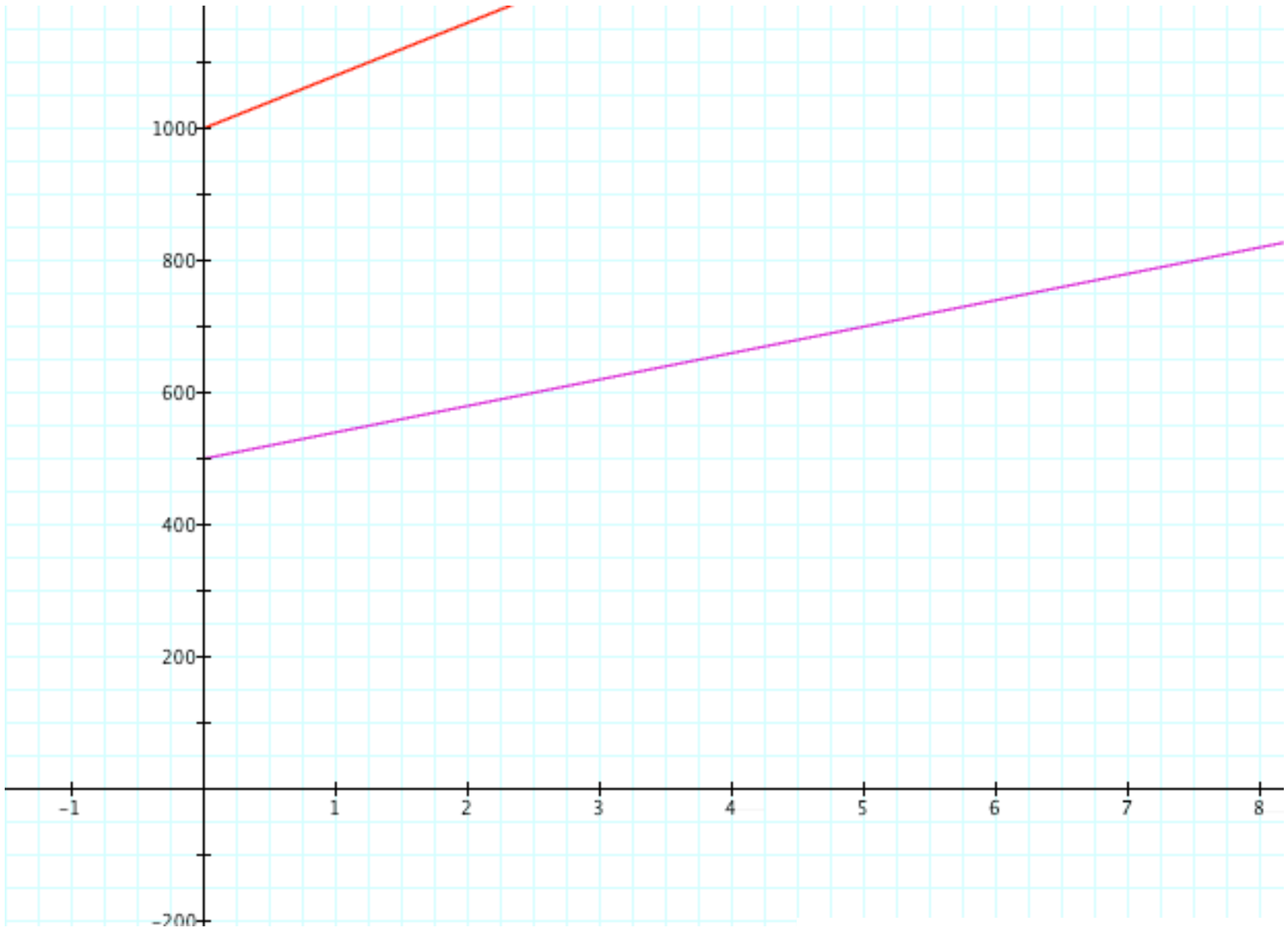


Function Family

A=0.08Nt+N

$q(x, n) = n + .08(n)x$

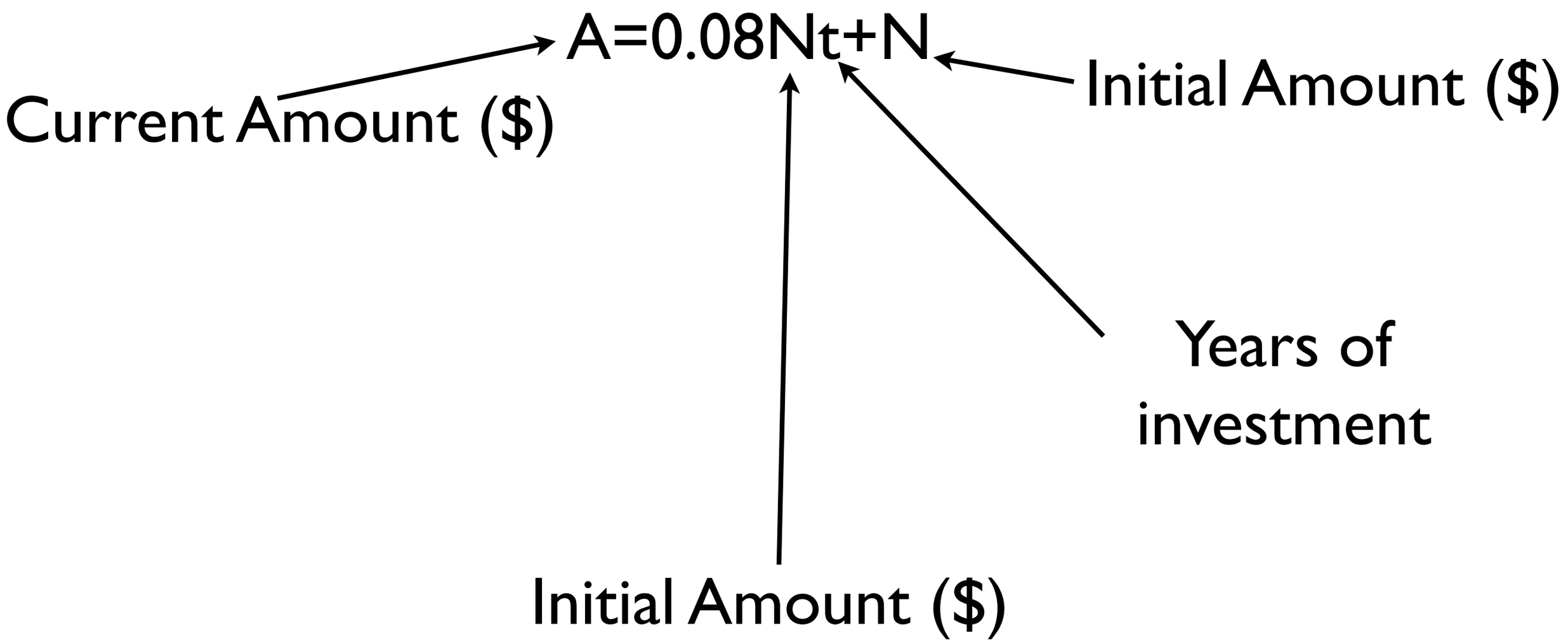
Function Family



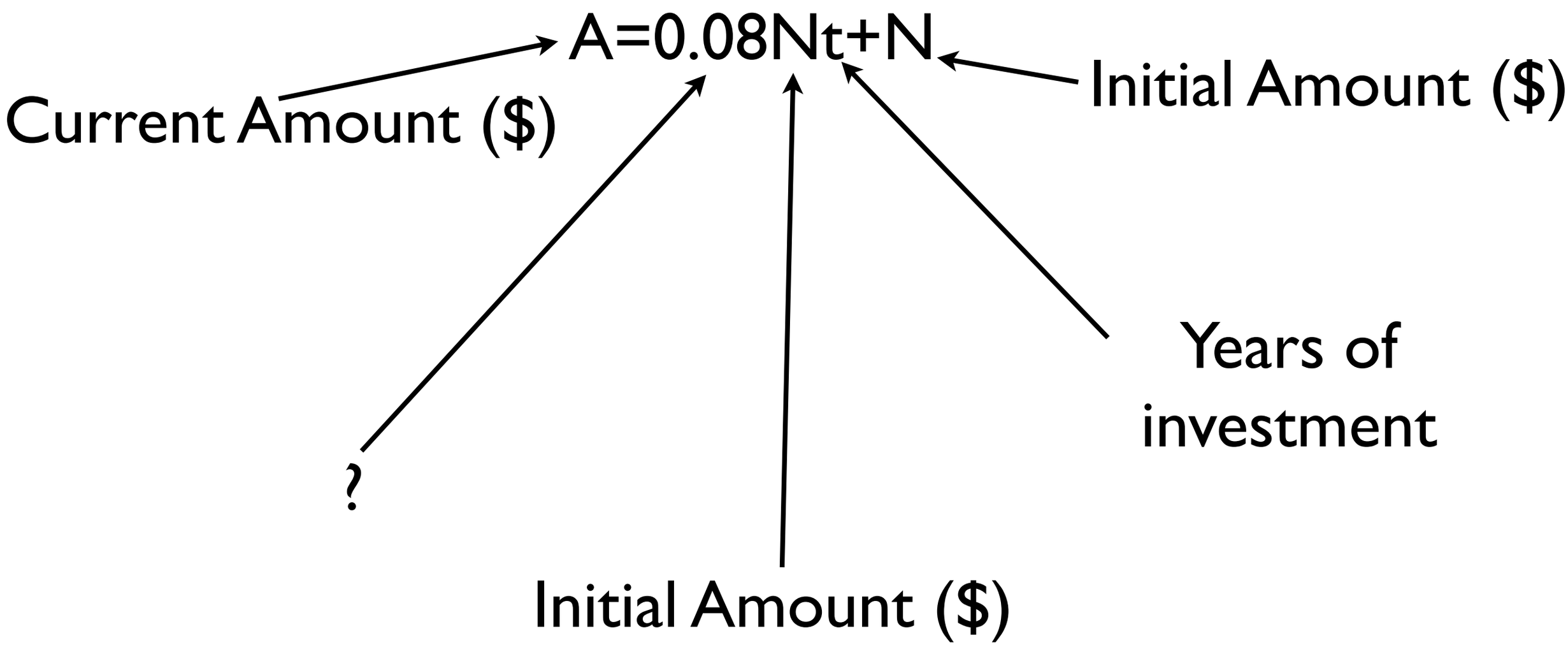
$A=0.08Nt+N$

$g(x,n) = n + .08(n)x$

Function Family



Function Family



What is 0.08?

$$A=0.08Nt+N$$

- A is a number of $.
- N is a number of $.
- 0.08Nt is a number of dollars.
- t is a number of years
- 0.08N is a number of dollars per year.
- 0.08 is a number of dollars per year per dollar

Simple Interest

Jodan bank uses a simple interest policy for their EZ8 investment accounts. The value of an EZ8 account grows at a rate of eight percent of the initial investment per year.

"For each 100"

Simple Interest

Jodan bank uses a simple interest policy for their EZ8 investment accounts. The value of an EZ8 account grows at a rate of eight percent of the initial investment per year.

"For each 100 dollars you invest, you earn 8 dollars per year."

8 dollars per year per 100 dollars.

0.08 dollars per year per dollar

Per Capita Rate of Change (1)

Patricia begins with $500 in her account

She has 500 dollar bills in her account.

Each dollar bill earns 0.08 dollars per year.

In total, she has $1+$1+$1+$1+$1....(500 times)=$500

In total, she earns at a rate of

0.08+0.08+0.08+...(500 times)=.0.08*500 dollars per year.

[Distributive Property]

Simple Interest

Jodan bank uses a simple interest policy for their EZ8 investment accounts. The value of an EZ8 account grows at a rate of eight percent of the initial investment per year.

What's the best strategy for investing with this account policy?

Compound Interest

The competing Yoi Trust has introduced a modification to Jodan's EZ8, which they call the YR8 account. Like the EZ8 account, the YR8 earns 8% of the initial investment per year. However, four times a year, Yoi Trust recalculates the "initial investment" of the YR8 account to include all the interest that the customer has earned up to that point.

Patricia begins with $500 in her account

Creating a function

- At time 0 years, Patricia has 500 dollars in her account.
- From time 0 to time 0.25, Patricia grows at a rate of 0.08*500 dollars per year.
- $P=0.08(500)t+500,\ 0 \leq t < 0.25$
- At 0.25 years, Patricia has 0.08*500*0.25+500 dollars
- [Distribute] Patricia has 500(1.08/4) dollars at 1/4 year.

Creating a function

- Patricia has 500(1.08/4) dollars at 1/4 year.
- At this time Yoi recalculates her interest rate. She now earns 0.08(500(1.08/4)) dollars per year.
- This lasts for the next 0.25 years.
- $P=0.08(500(1.08/4))(t-0.25) + 500(1.08/4), 0.25 \leq t<0.5$
- At the end of the quarter she has \$0.08(500(1.08/4))(0.25) + 500(1.08/4)
- [Distribute] $500(1.08/4)^2$ dollars at 0.5 years.

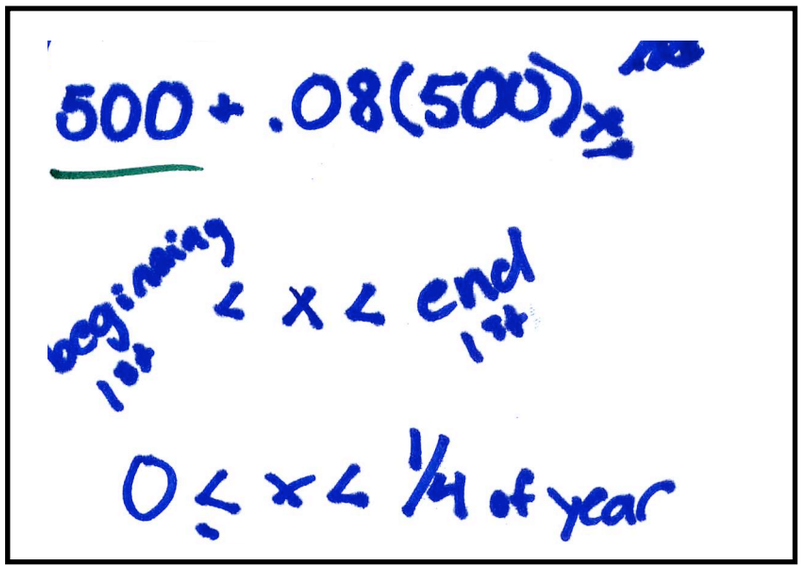
Creating a function

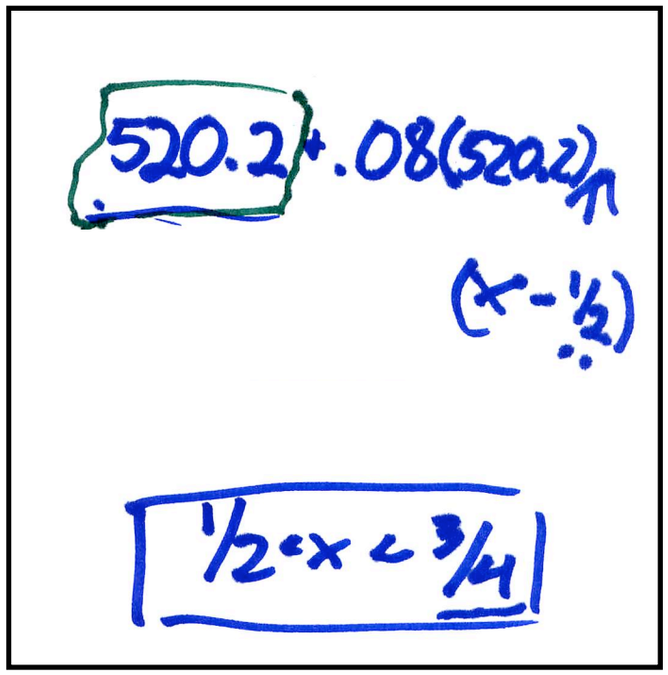
- $500(1.08/4)^2$ dollars at 0.5 years.
- Recalculate the interest rate for the next quarter
- $P=0.08\ 500(1.08/4)^2(t-0.5)+ 500(1.08/4)^2, 0.5 \leq x < 0.75$
- At 0.75 years, Patricia has $\$500(1.08/4)^3$
- $P=0.08\ 500(1.08/4)^3(t-0.75)+ 500(1.08/4)^3, 0.75 \leq x < 1$
- etc...

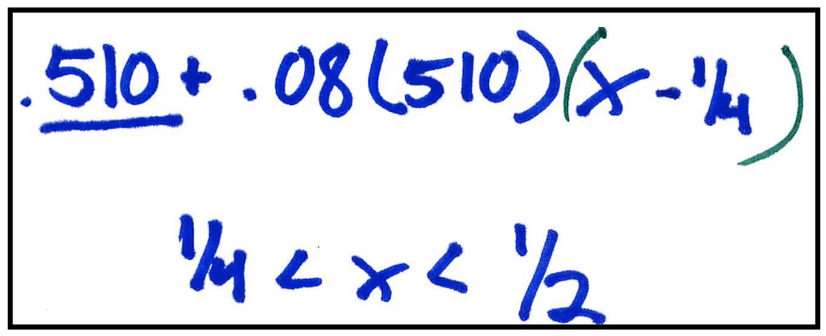
Our Function

- $P=0.08(500)t+500,\ 0 \leq t<0.25$
- $P=0.08(500(1.08/4))(t-0.25) + 500(1.08/4),\ 0.25 \leq t<0.5$
- $P=0.08\ 500(1.08/4)^2(t-0.5)+ 500(1.08/4)^2,\ 0.5 \leq x<0.75$
- $P=0.08\ 500(1.08/4)^3(t-0.75)+ 500(1.08/4)^3,\ 0.75 \leq x<1$
- ....

Our Function







This is complicated, but not impossible.
Our students have the tools, but never learn how to use them.

Our Function

$500(1+\frac{.08}{4})^0 + .08(500)x$

$500(1+\frac{.08}{4})^1 + .08(510)(x-\frac{1}{4})$

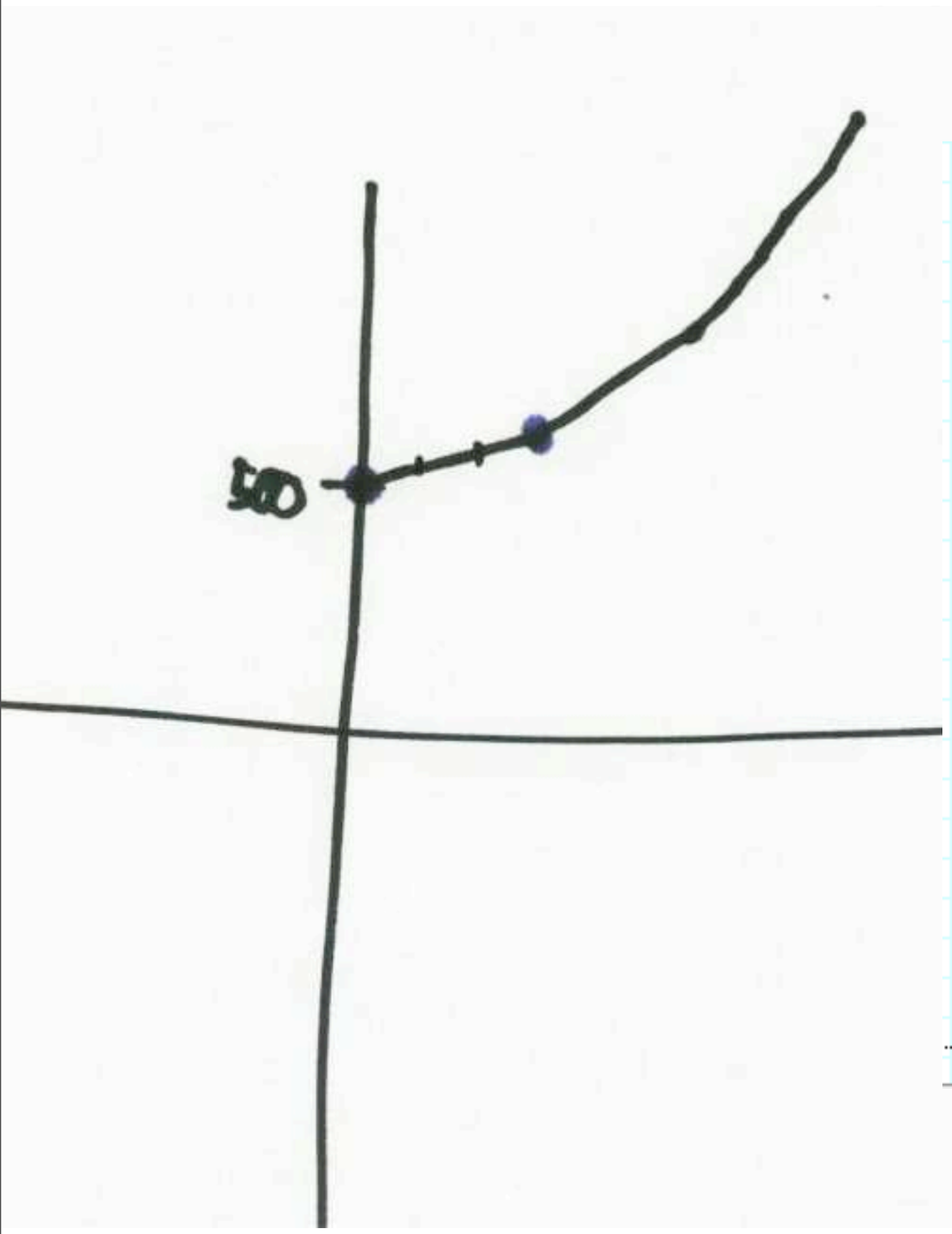
$500(1+\frac{.08}{4})^2 + .08(520.2)(x-\frac{1}{2})$

$500(1+\frac{.08}{4})^5 + .08(500(1+\frac{.08}{4})^5)(x-\frac{5}{4})$

$\frac{5}{4} < x < \frac{6}{4}$

What would a graph look like?

Our Function



Three versions of the compound interest formula

Traditional $$A(x) = P\left(1 + \frac{r}{n}\right)^{nx}$$

Piecewise Linear $$A(x) = P\left(1 + \frac{r}{n}\right)^{\lfloor nx \rfloor} + r\left(1 + \frac{r}{n}\right)^{\lfloor nx \rfloor}\left(x - \frac{\lfloor nx \rfloor}{n}\right)$$

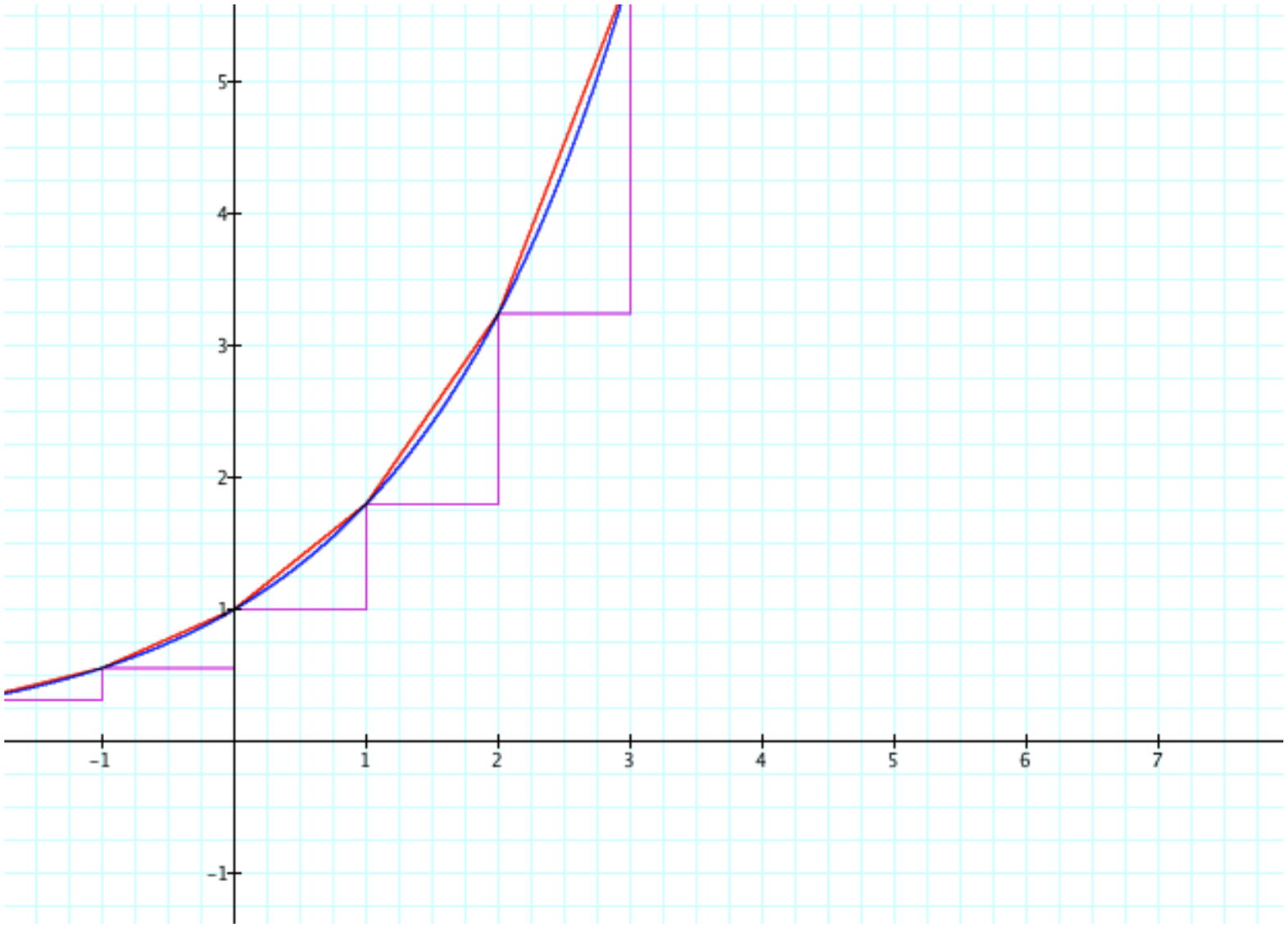
Actual $$A(x) = P\left(1 + \frac{r}{n}\right)^{\lfloor nx \rfloor}$$

Three versions of the compound interest formula

Traditional

Piecewise Linear

Actual



Limit

But what is at that limit really?

- Simple Interest: 0.08 dollars per initial dollar per year
- Compound Interest: 0.08 dollars per dollar at the beginning of the compounding interval per year.
- Limit: 0.08 dollars per current dollar per year.

But what is at that limit really?

- Simple Interest: 0.08 dollars per initial dollar per year
- Compound Interest: 0.08 dollars per dollar at the beginning of the compounding interval per year.
- Limit: 0.08 dollars per current dollar per year. (Constant per-capita rate of change)

Using per-capita rate to calculate $/yr rate

- r=$/yr rate; y(t)=$in account at time *t*.
- Simple interest r=0.08y(0)
- Compound interest r=0.08y(n), n=0,0.25,0.5,0.75,1,1.25,....
- Limit r=0.08y(t)

Constant Per-Capita Rate of change

- As *r* is changing, and *y* is changing
- $r=0.08y$
- where *r* is the rate of change of *y* with respect to *t*.

Grandchildren

- Each dollar is earning 8 cents per year (constant rate for each dollar.).
  - In simple interest, only my original dollars earn 8 cents per year.
    - The ‘child’ dollars do not earn interest
    - my $/year rate stays constant.
  - In constant rate of change. Every dollar makes 8 cents per year.
    - As new ‘dollars’ are born, they also make new dollars.
    - Those children and grandchildren and great-grand children all contribute to the rate!
    - They don’t wait to be born either. Even a partially complete dollar is making new dollars (just slower).

Constant per-capita rate of change

The Savings Company (SayCo) also competes with Yoi Trust and Jodan. SayCo's PD8 account policy is as follows: if you have one dollar in your bank account, you earn interest at a rate of 8 cents per year. For each additional dollar, your interest increases by another 8 cents per year. If you have fractions of a dollar in your account, your interest increases by the same fraction, so 50 cents earns interest at 4 cents per year.

Constant per-capita rate of change

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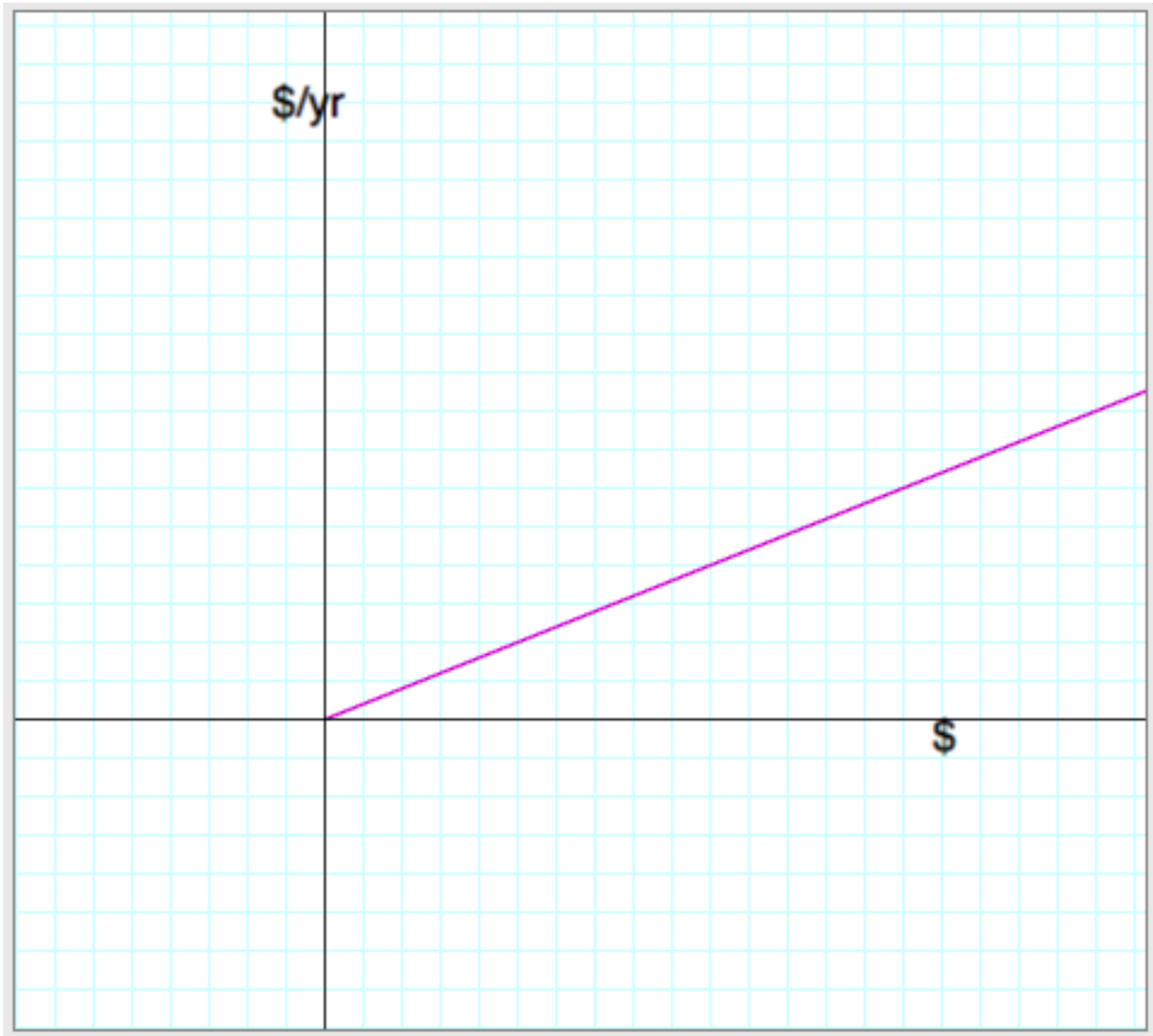
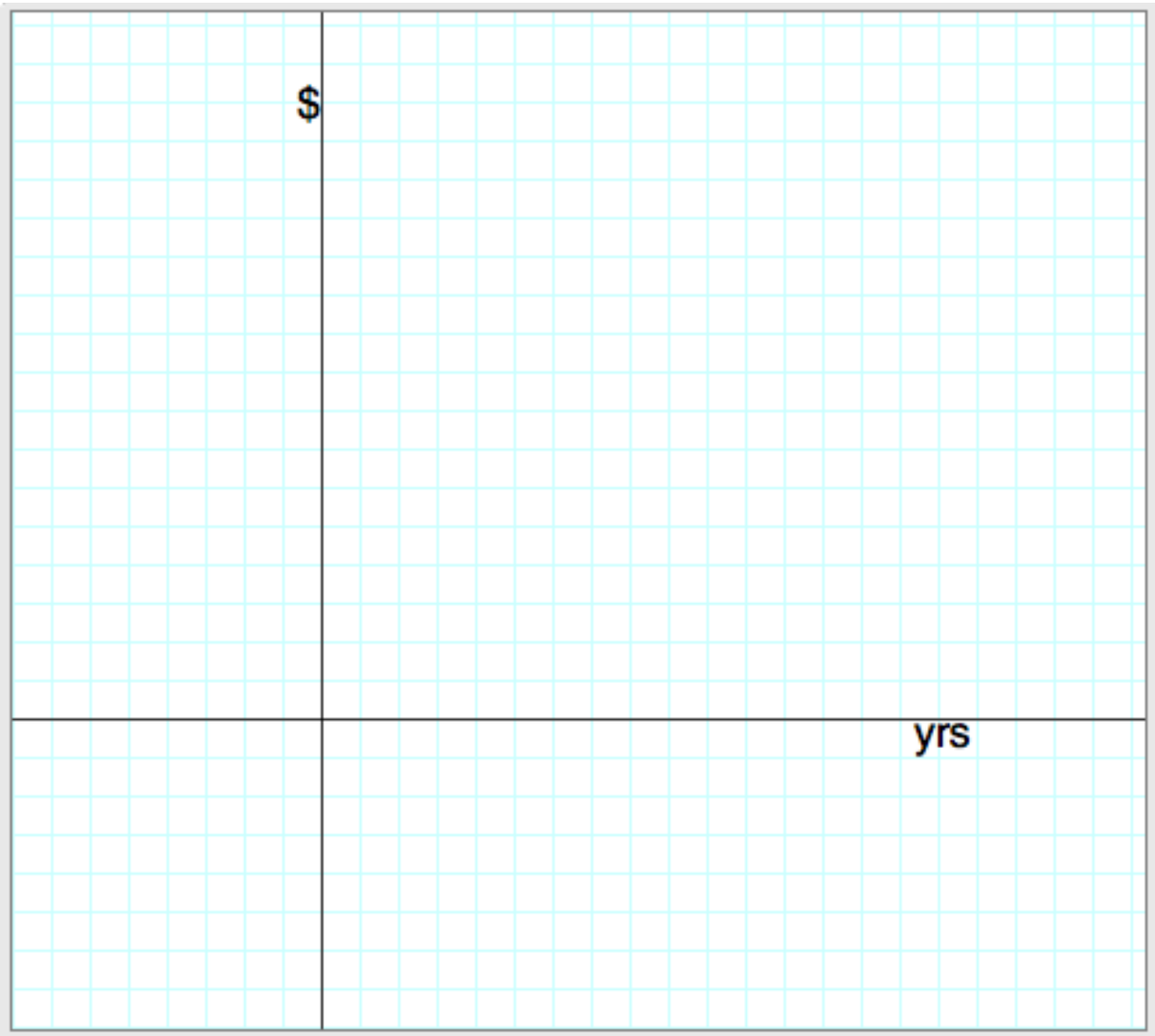
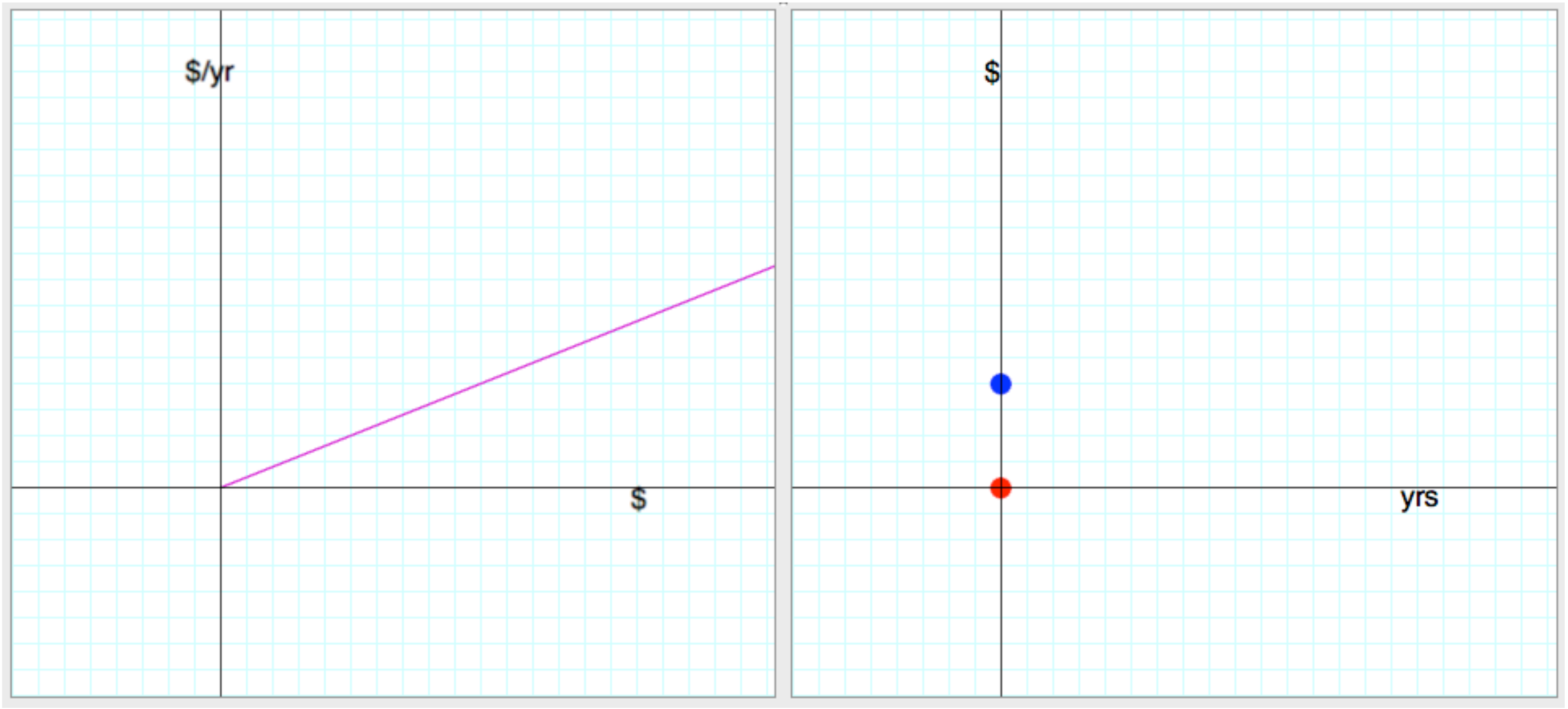
Key ideas:

- There is no updating your rate.
- At any time, whatever money you have at that time determines how fast you're earning money.
- $r=0.08y$


$/yr
$


$
yrs

Tiffany



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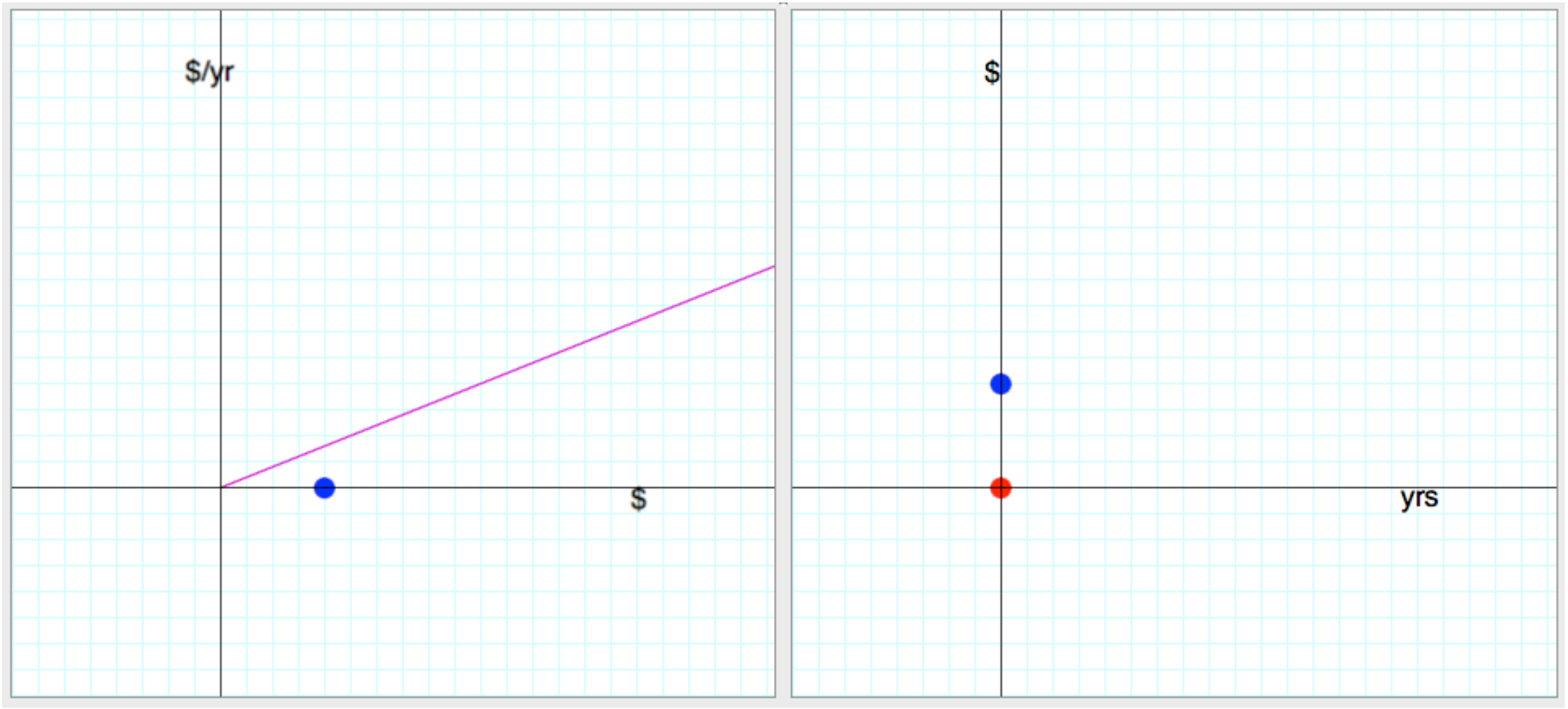

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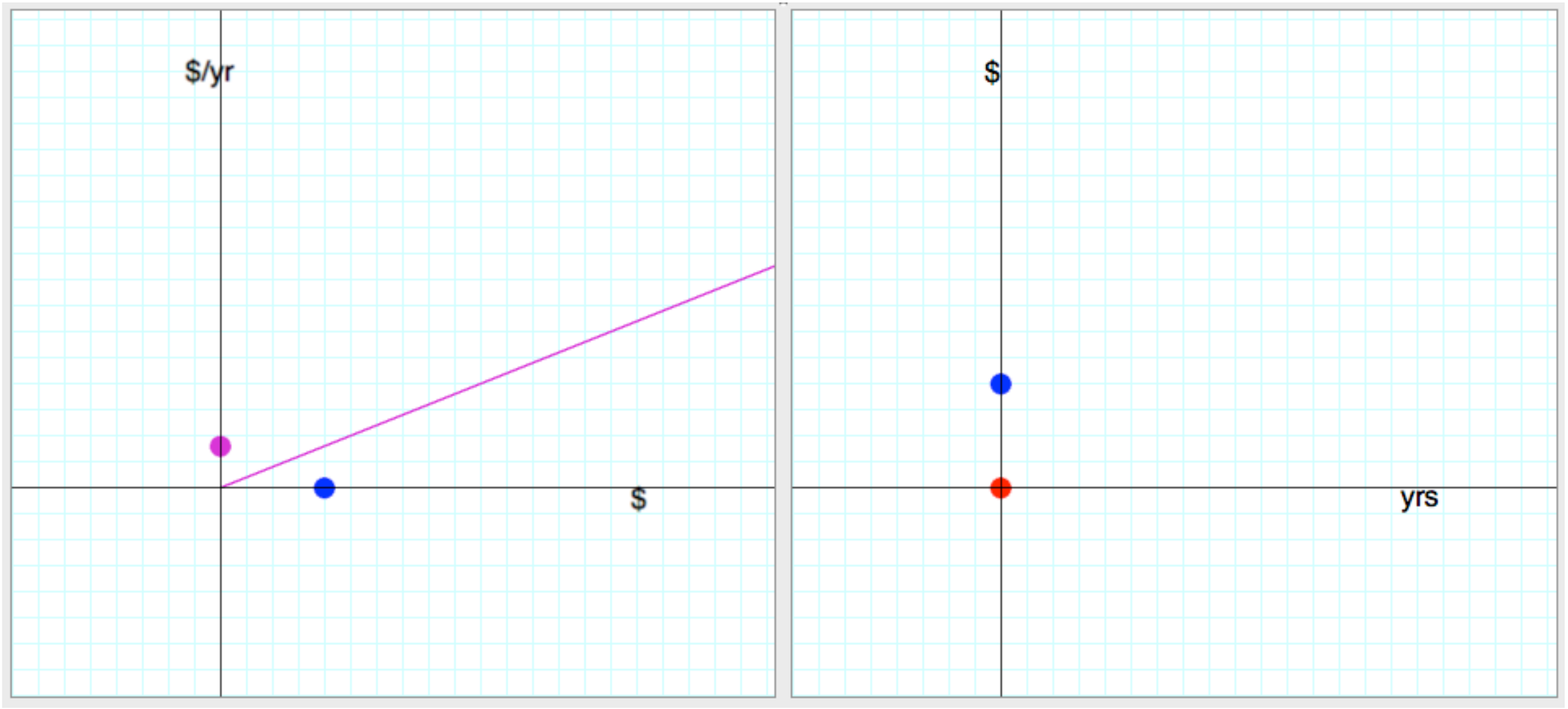

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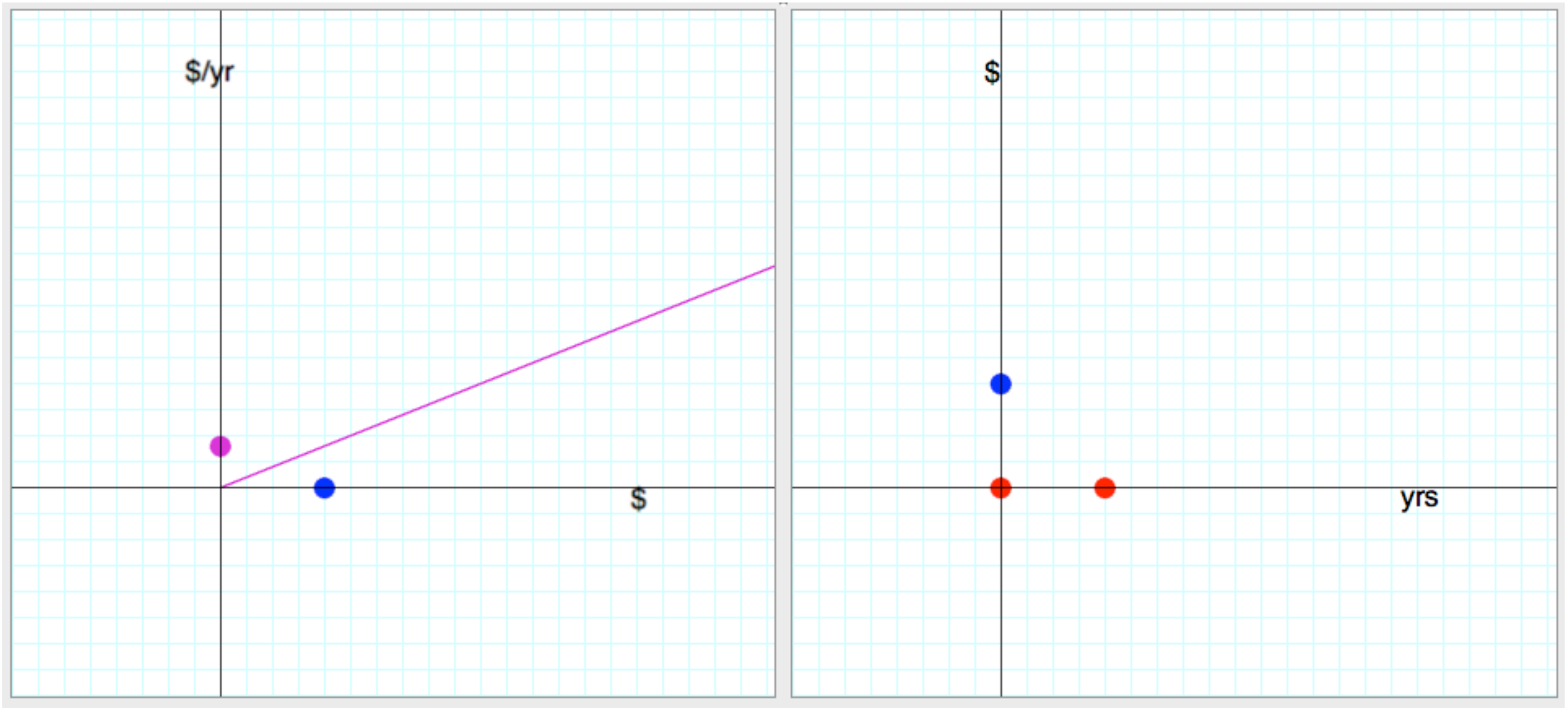

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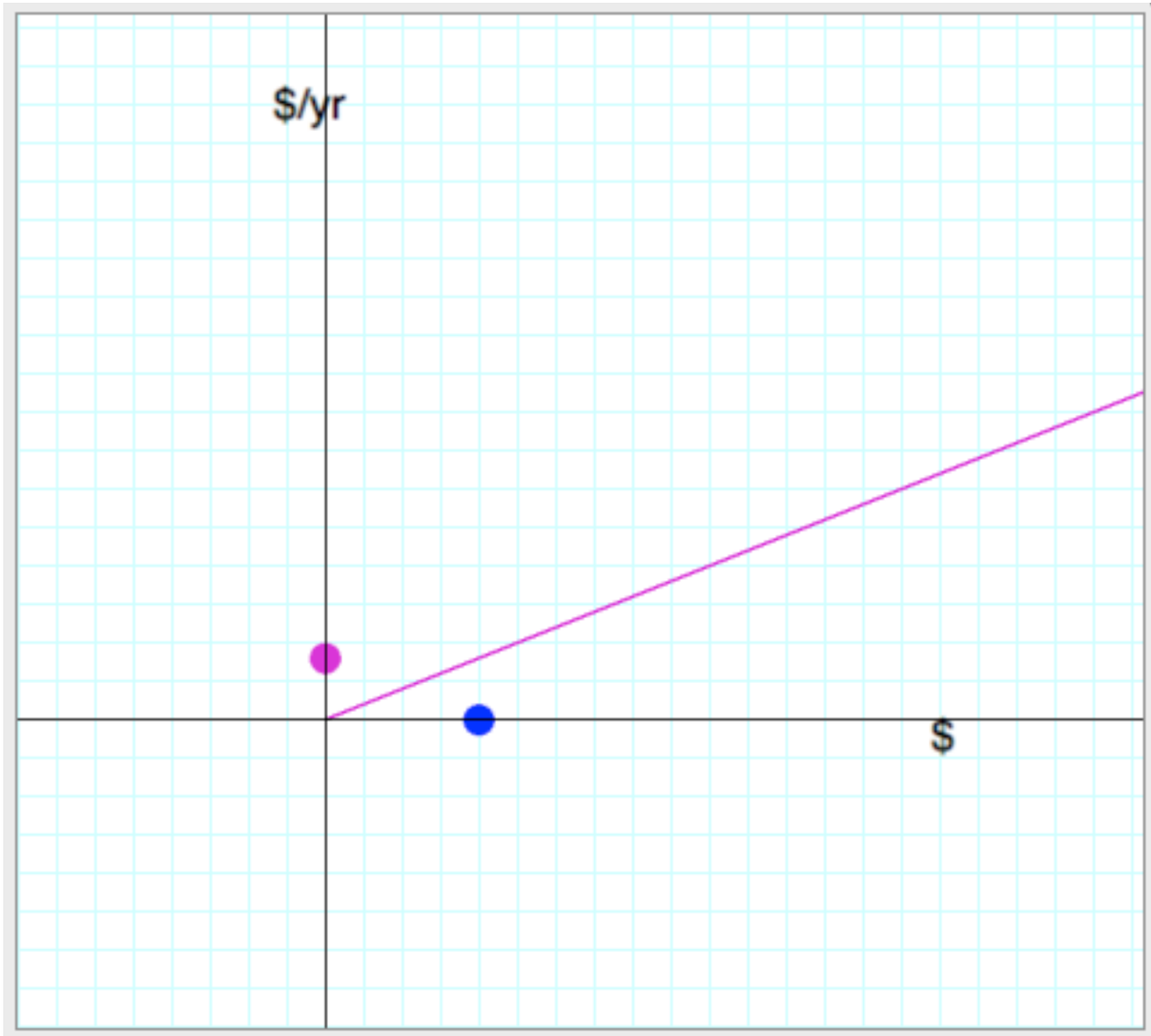

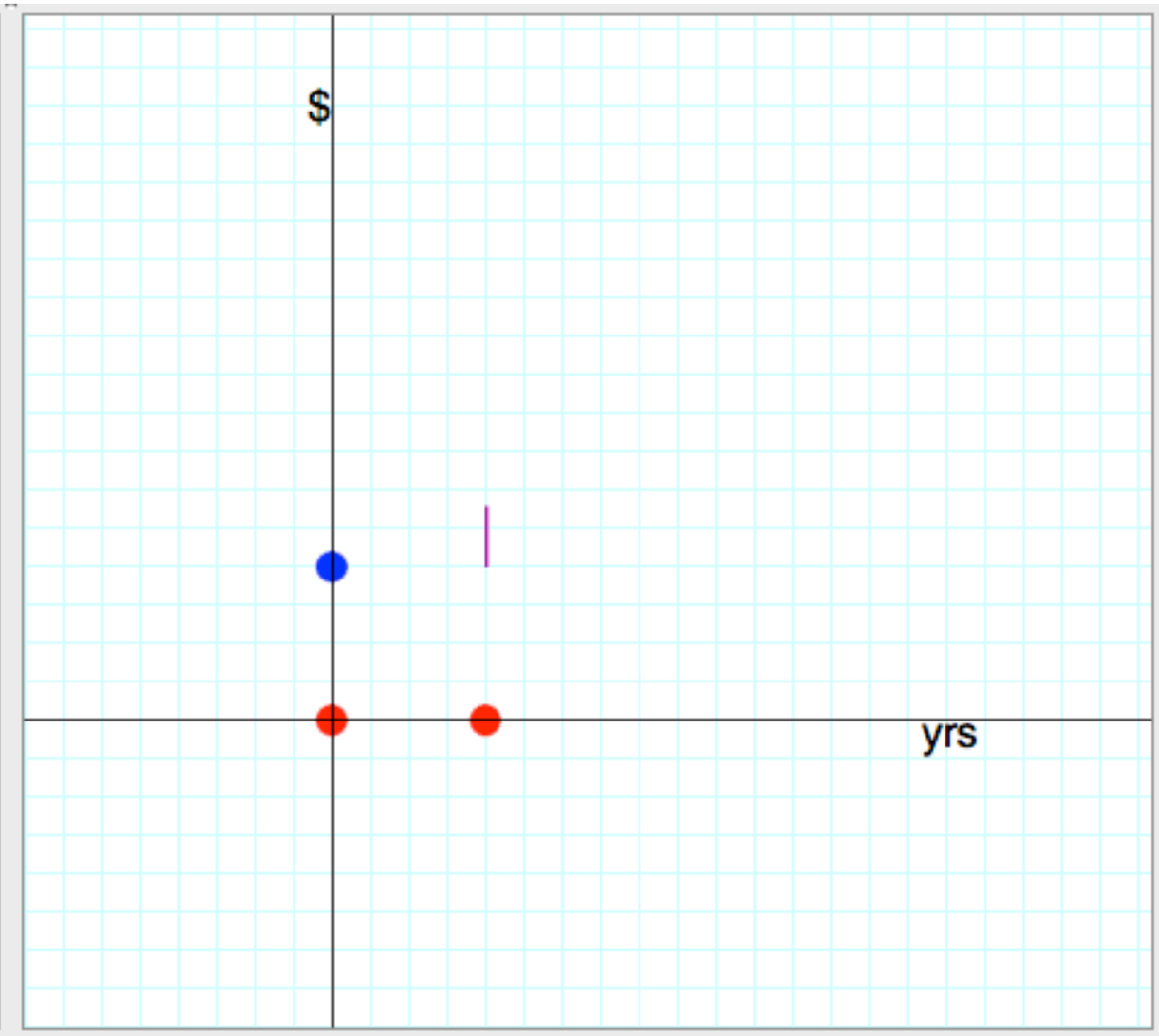

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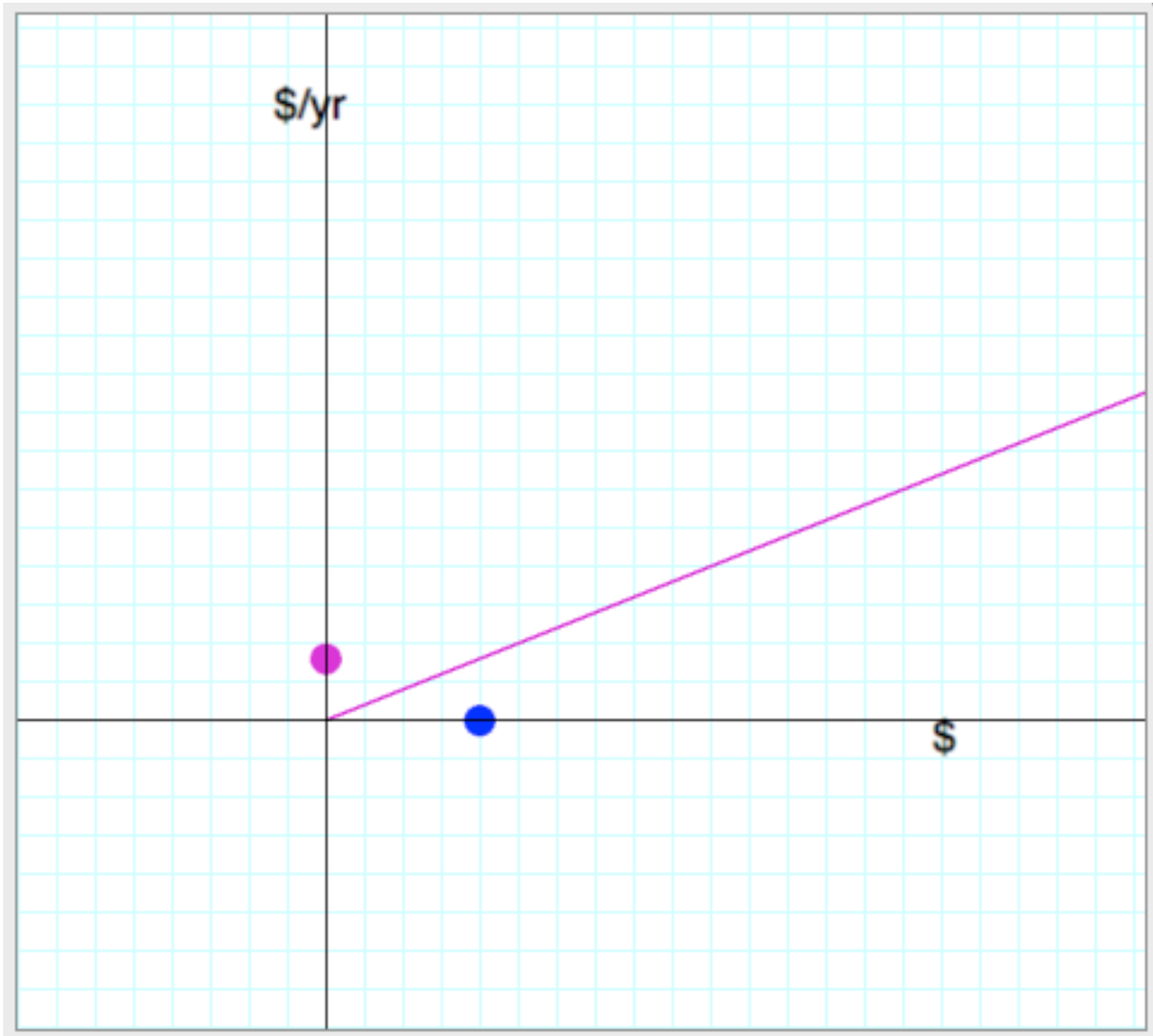

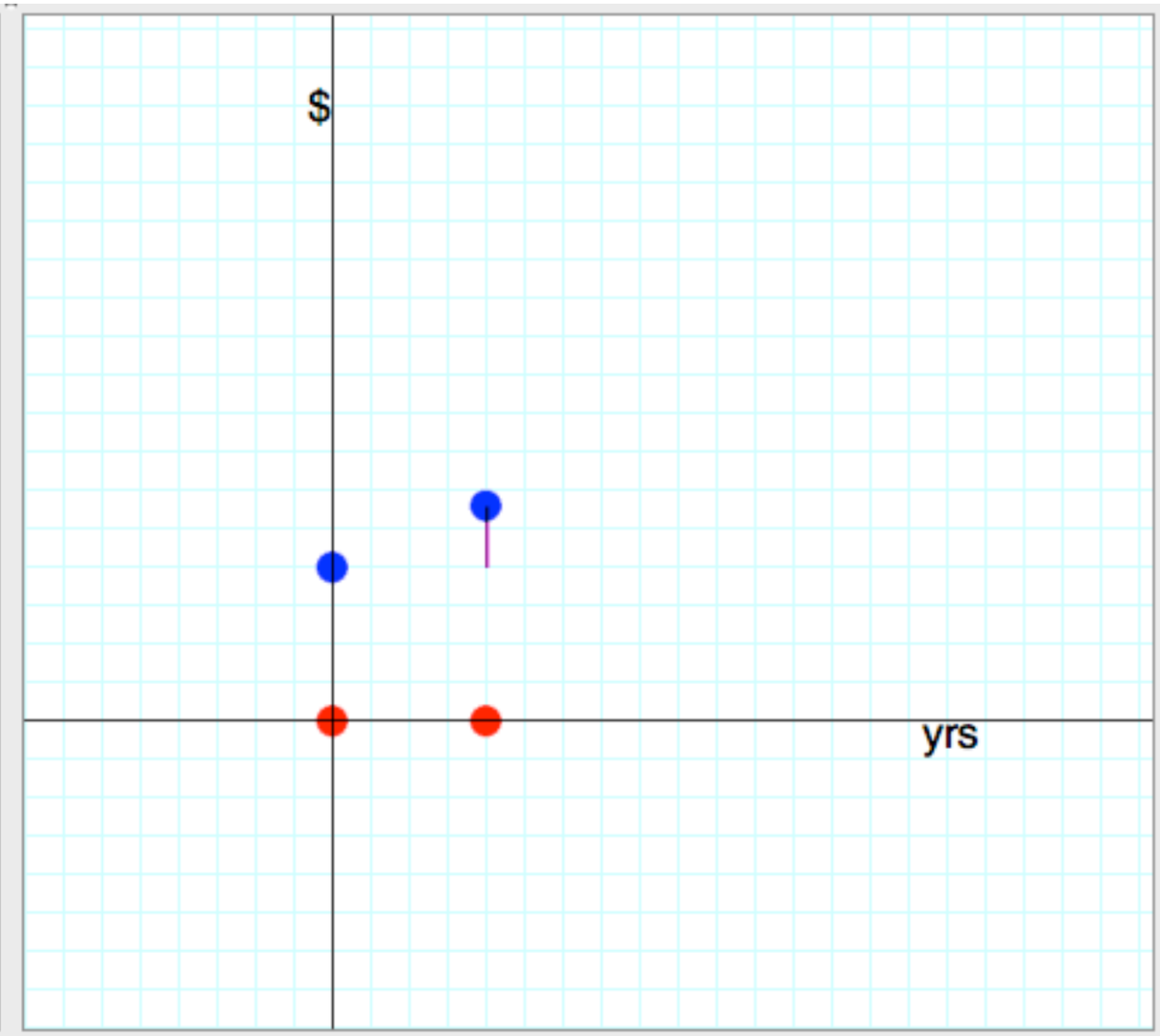

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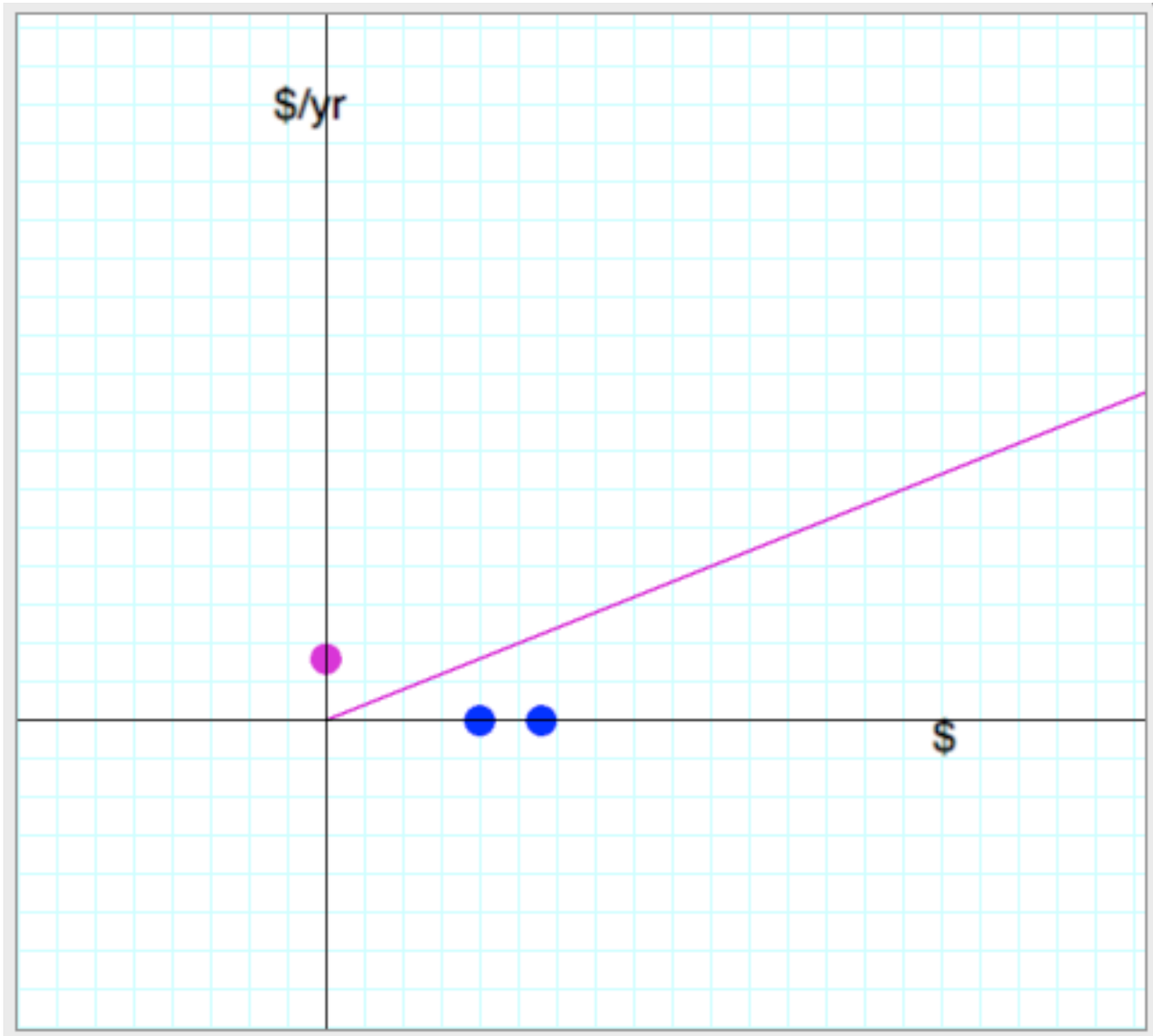

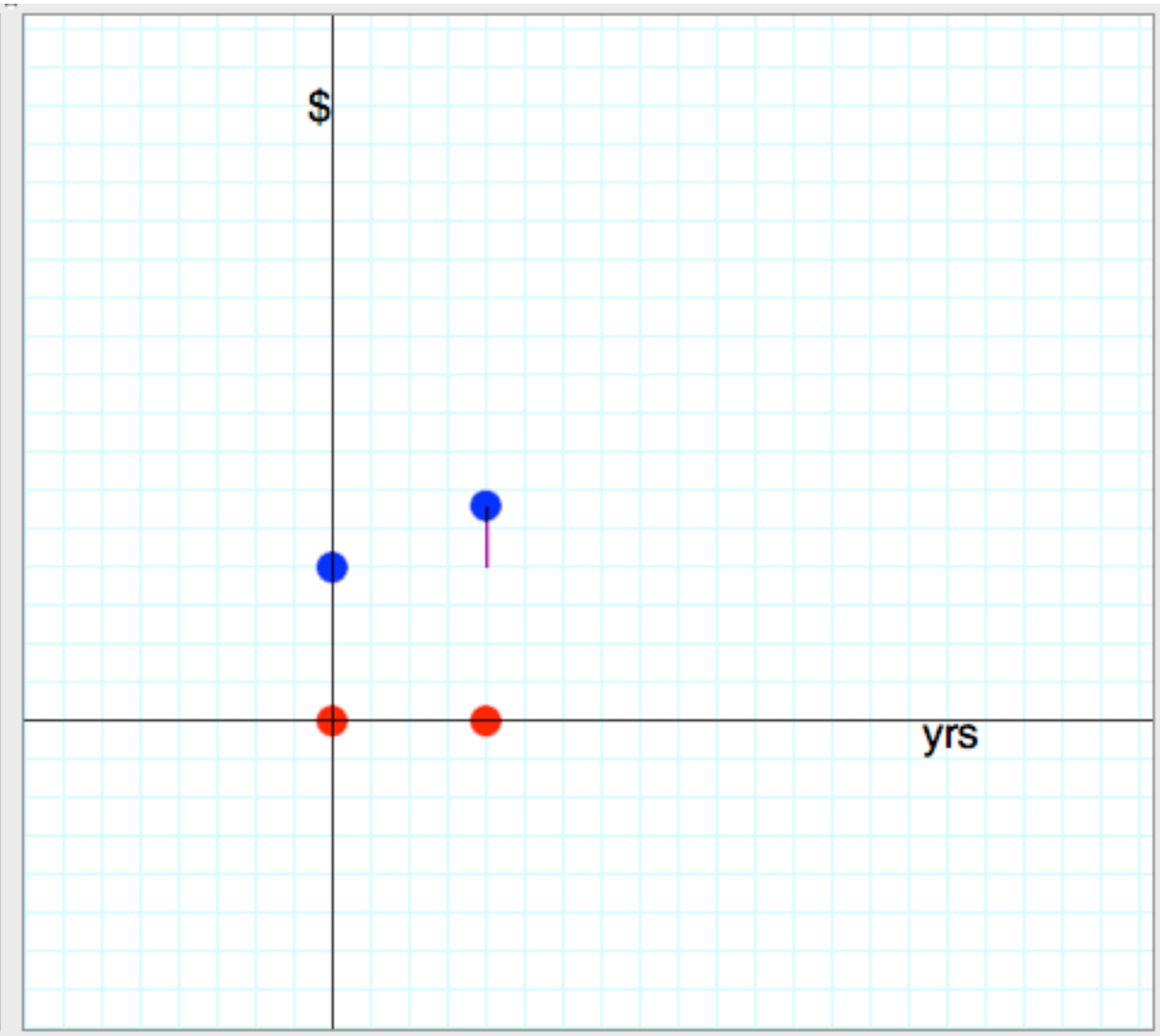

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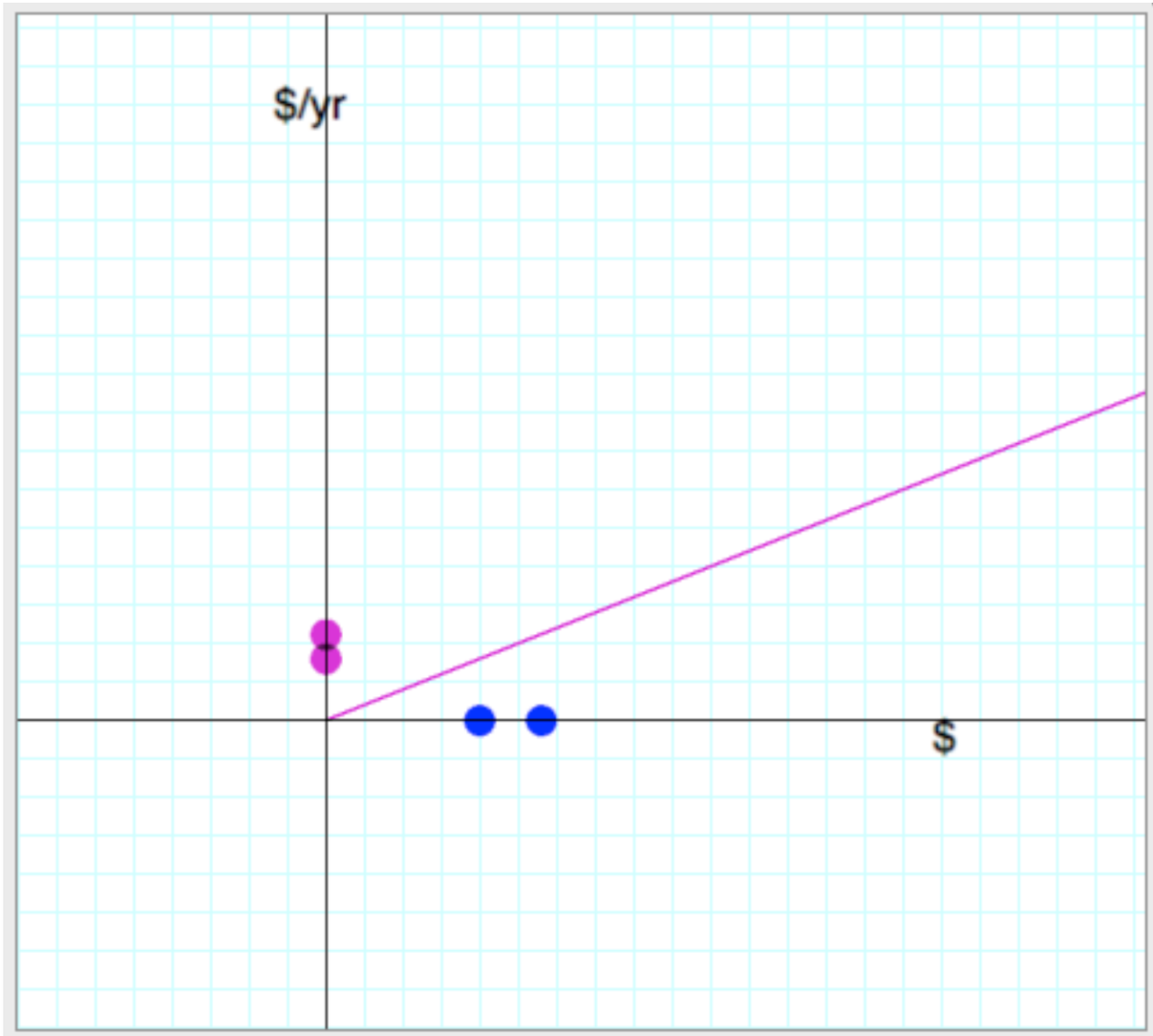

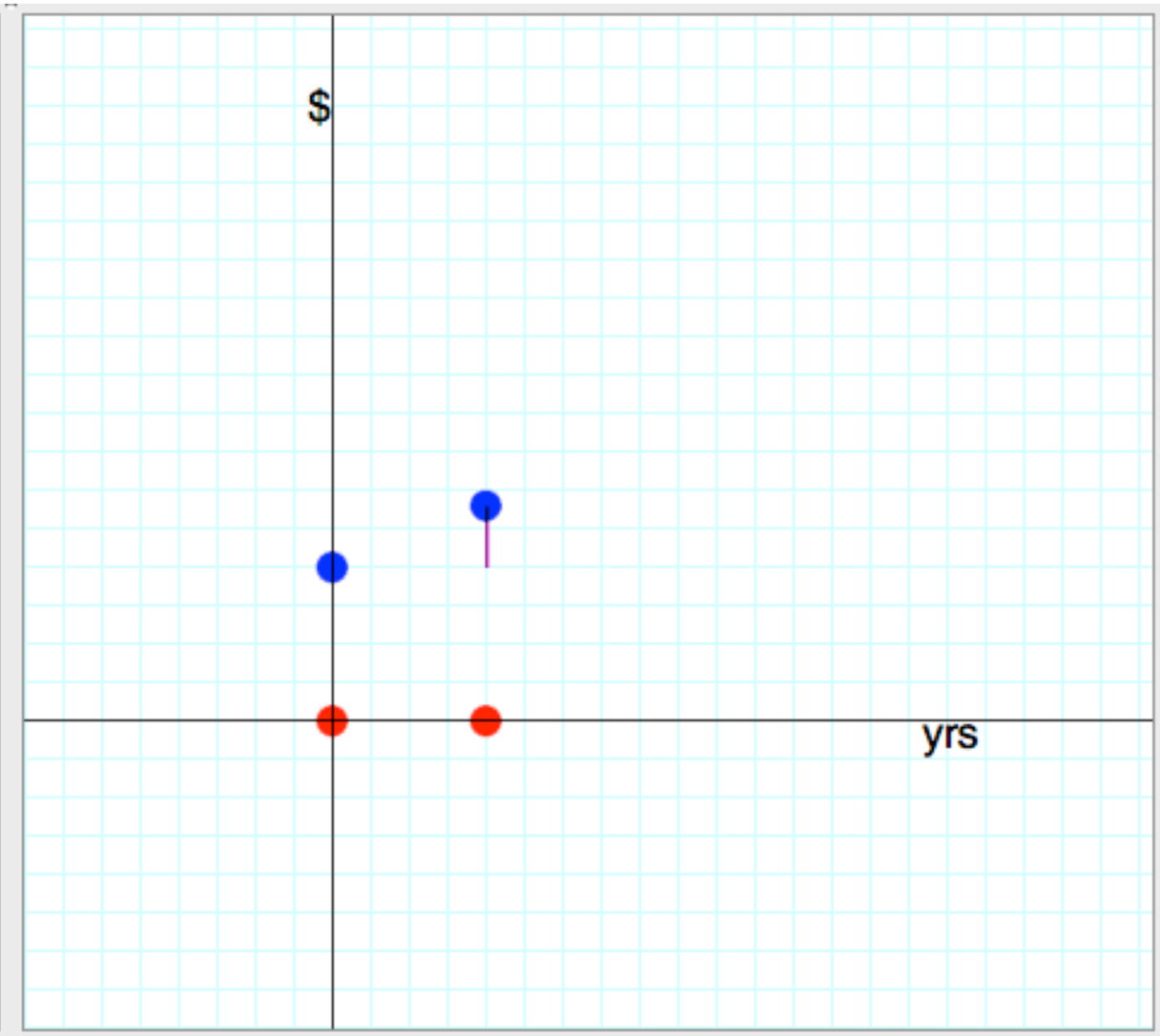

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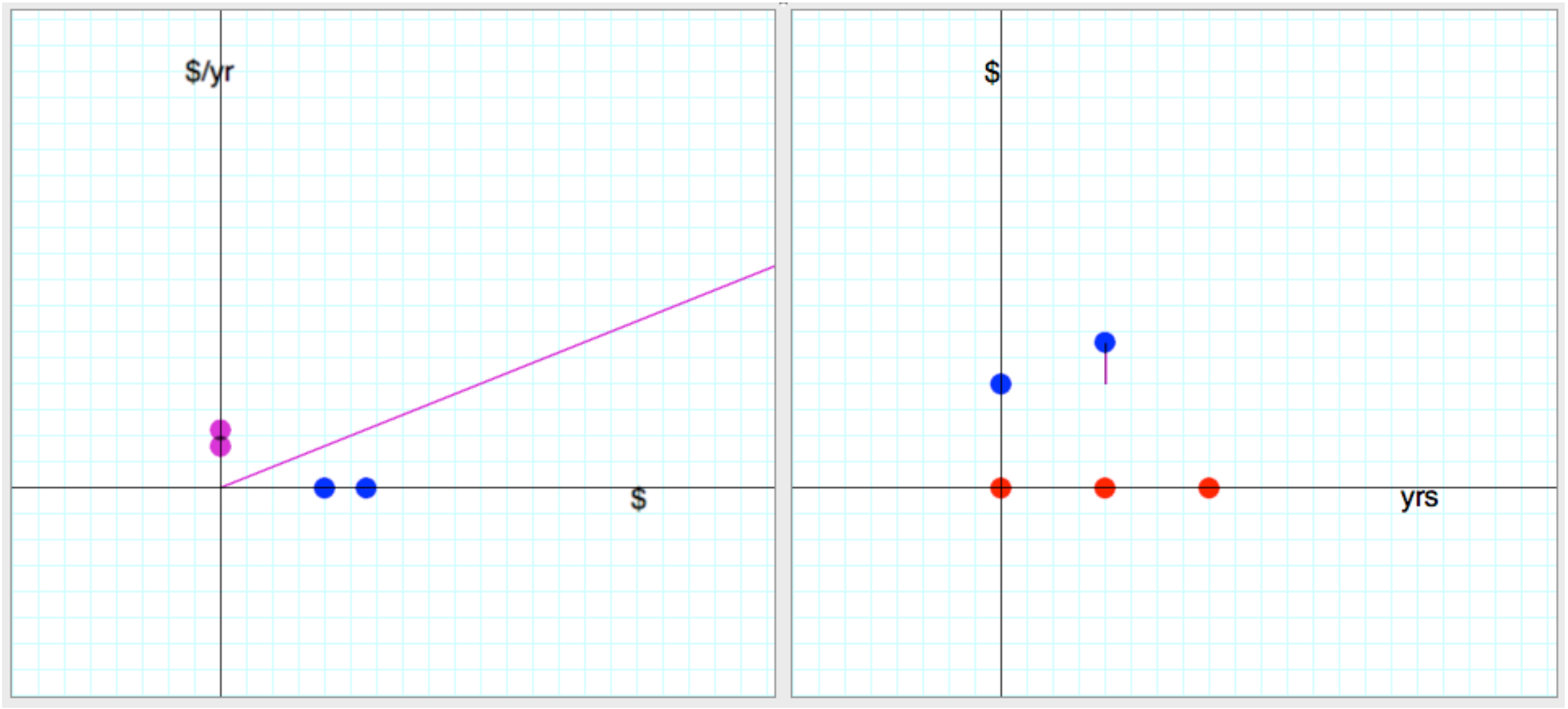

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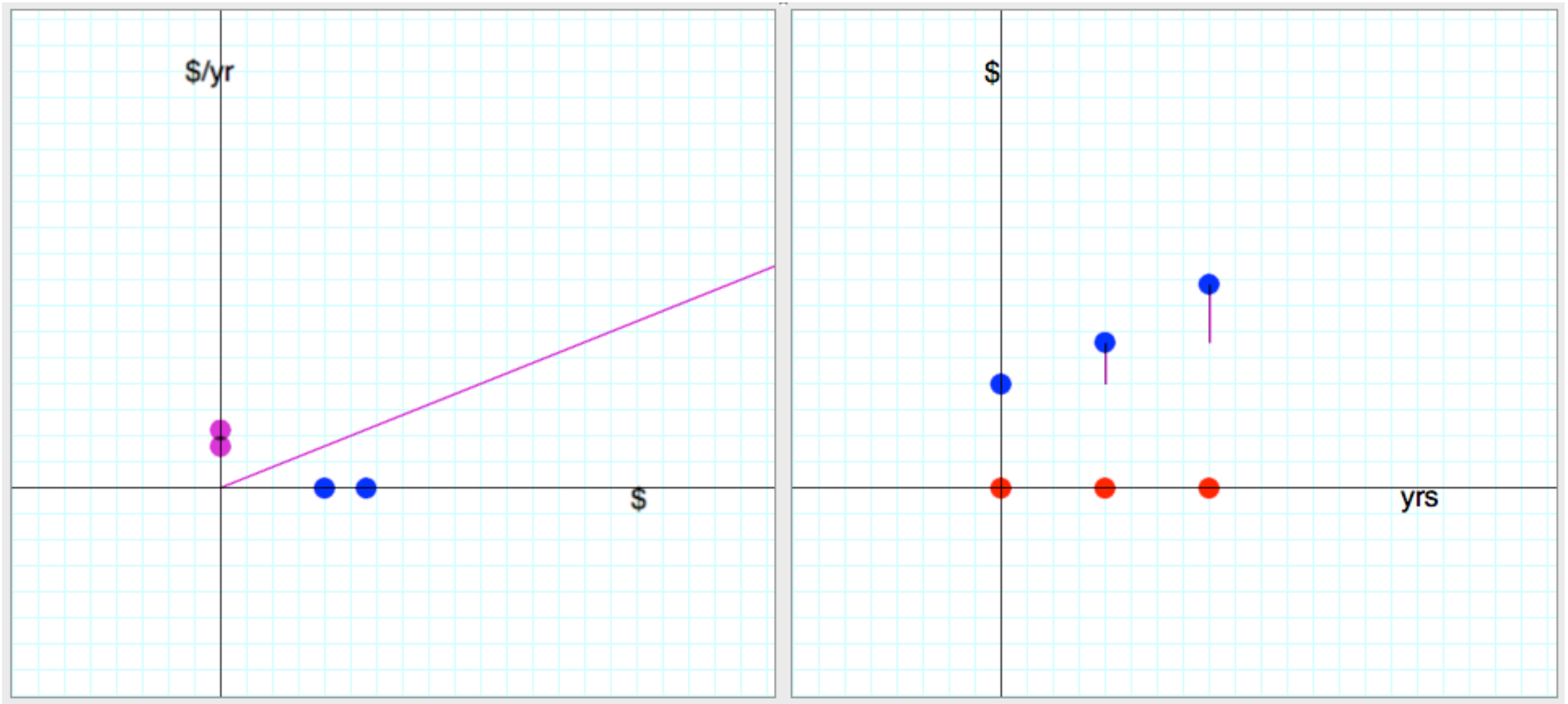

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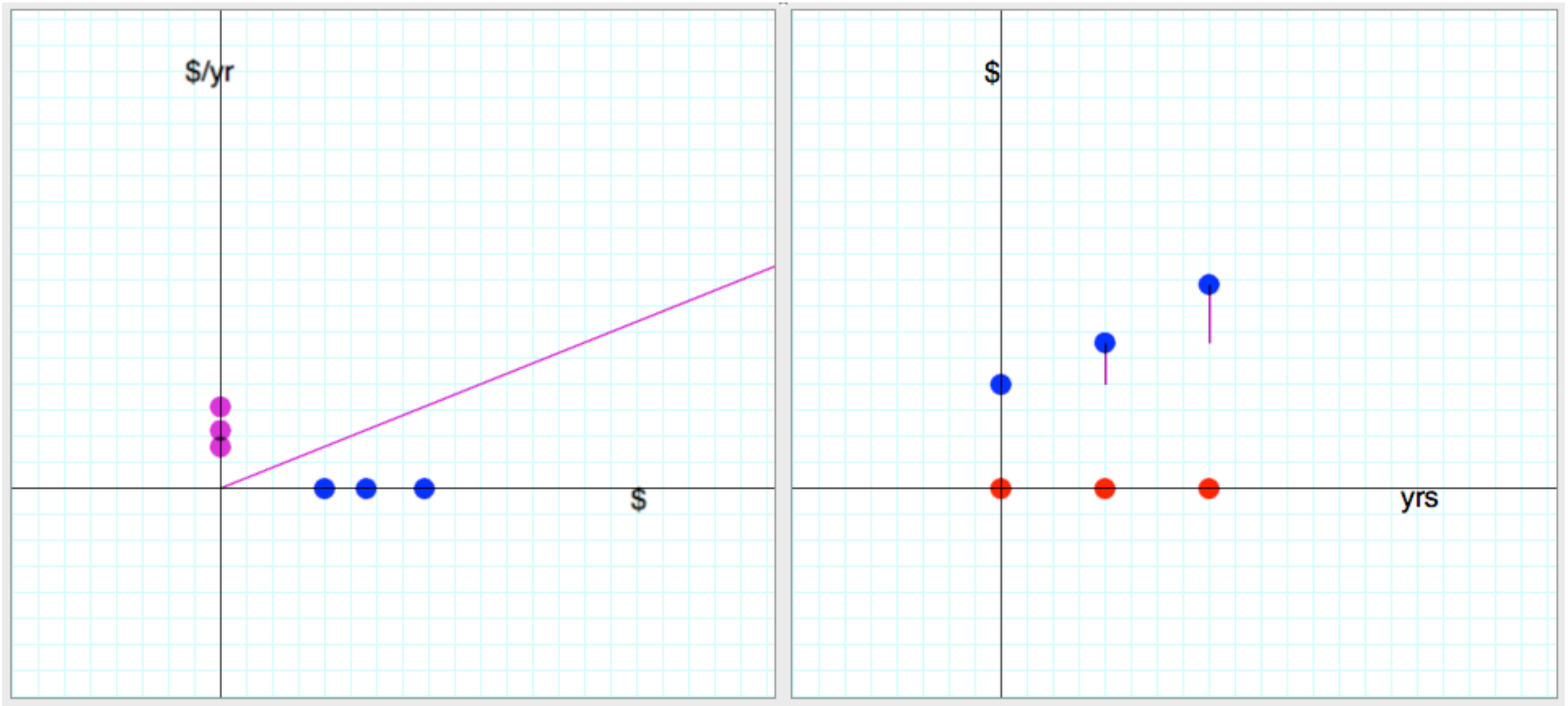

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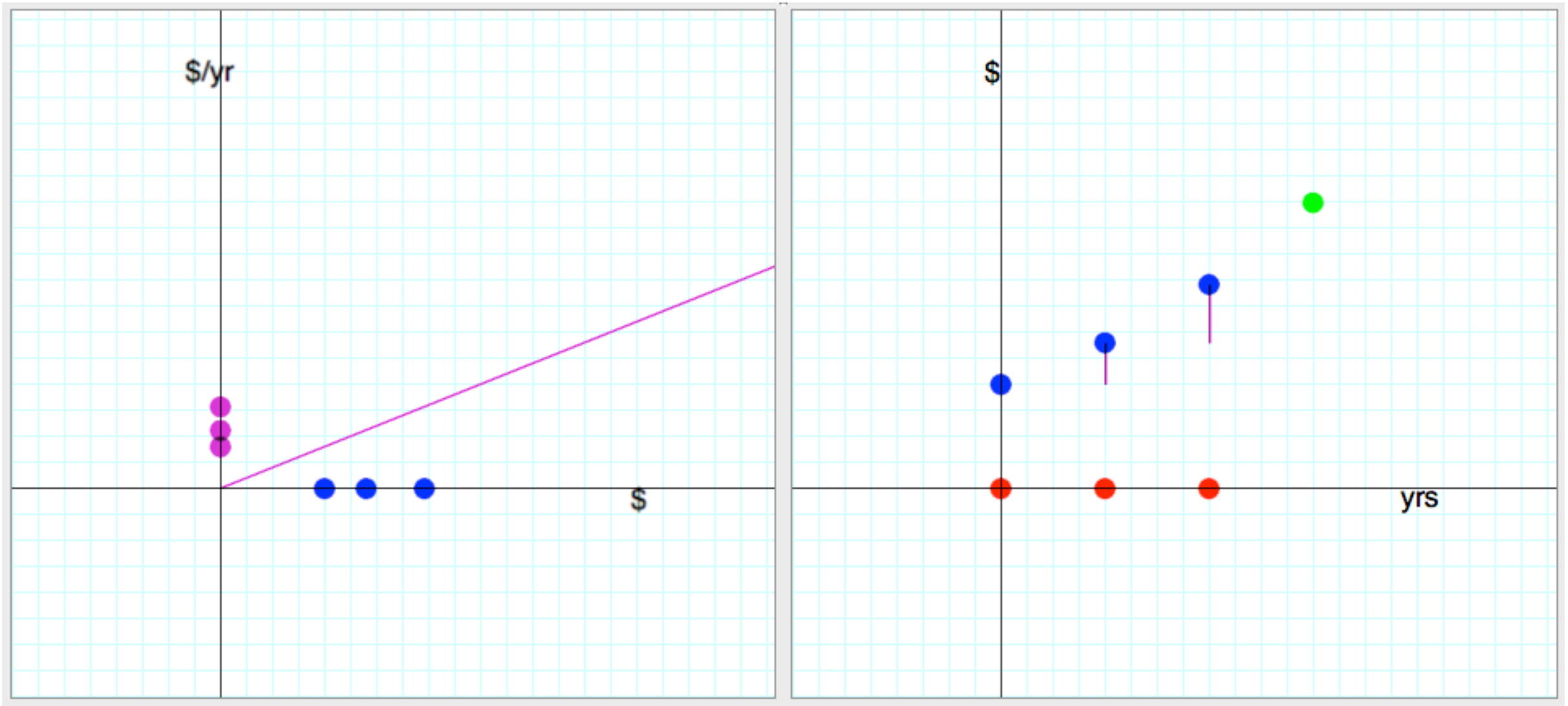

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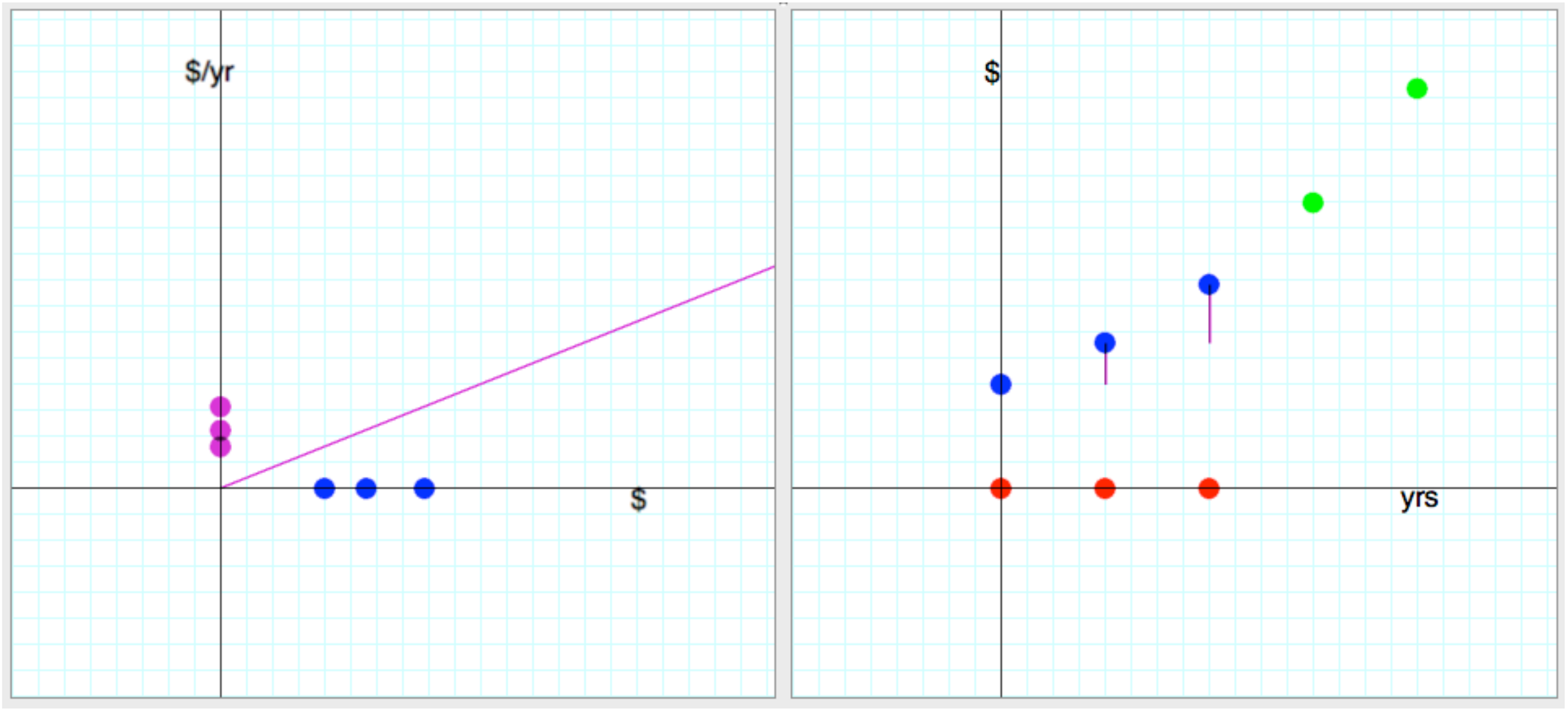

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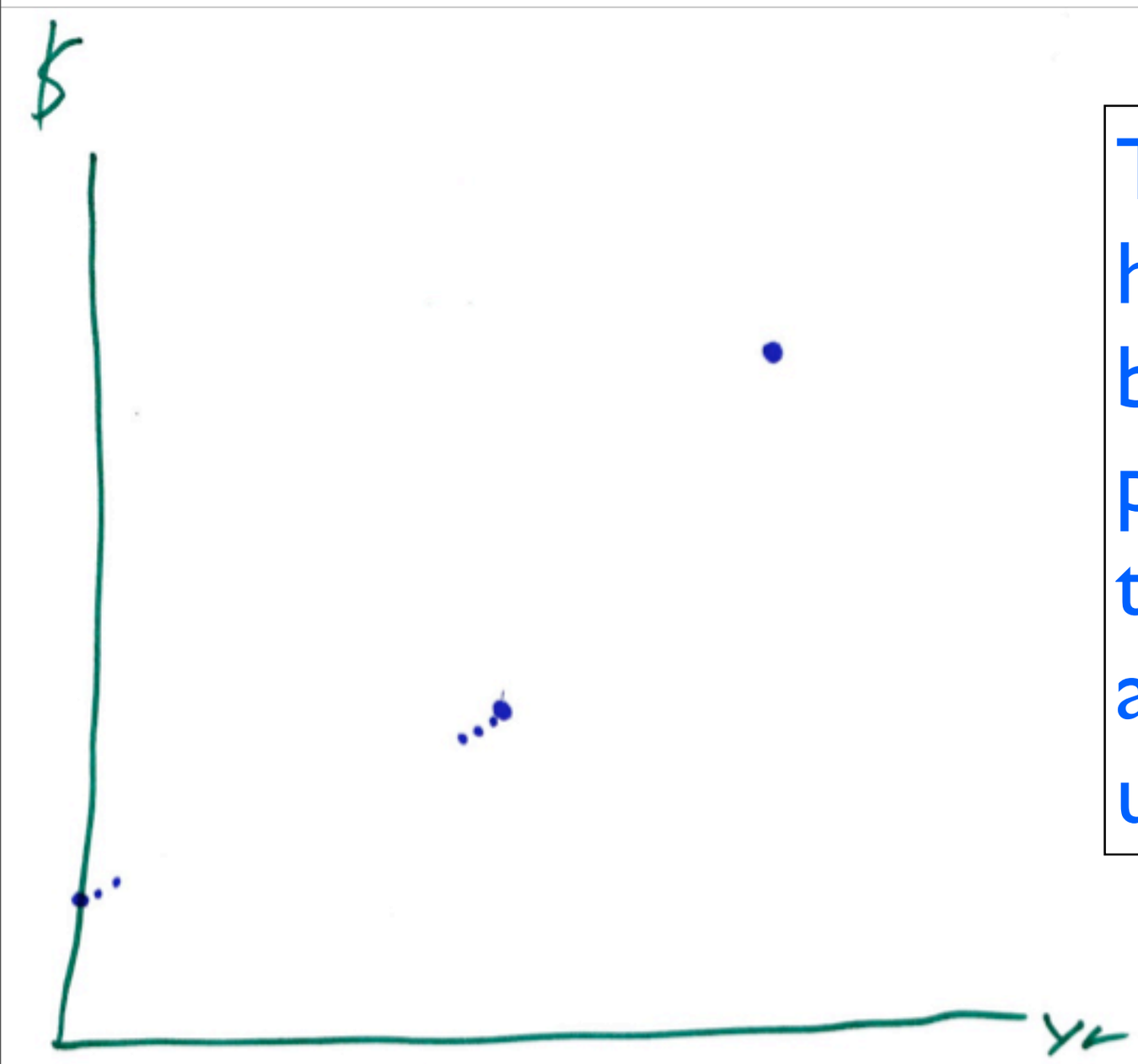
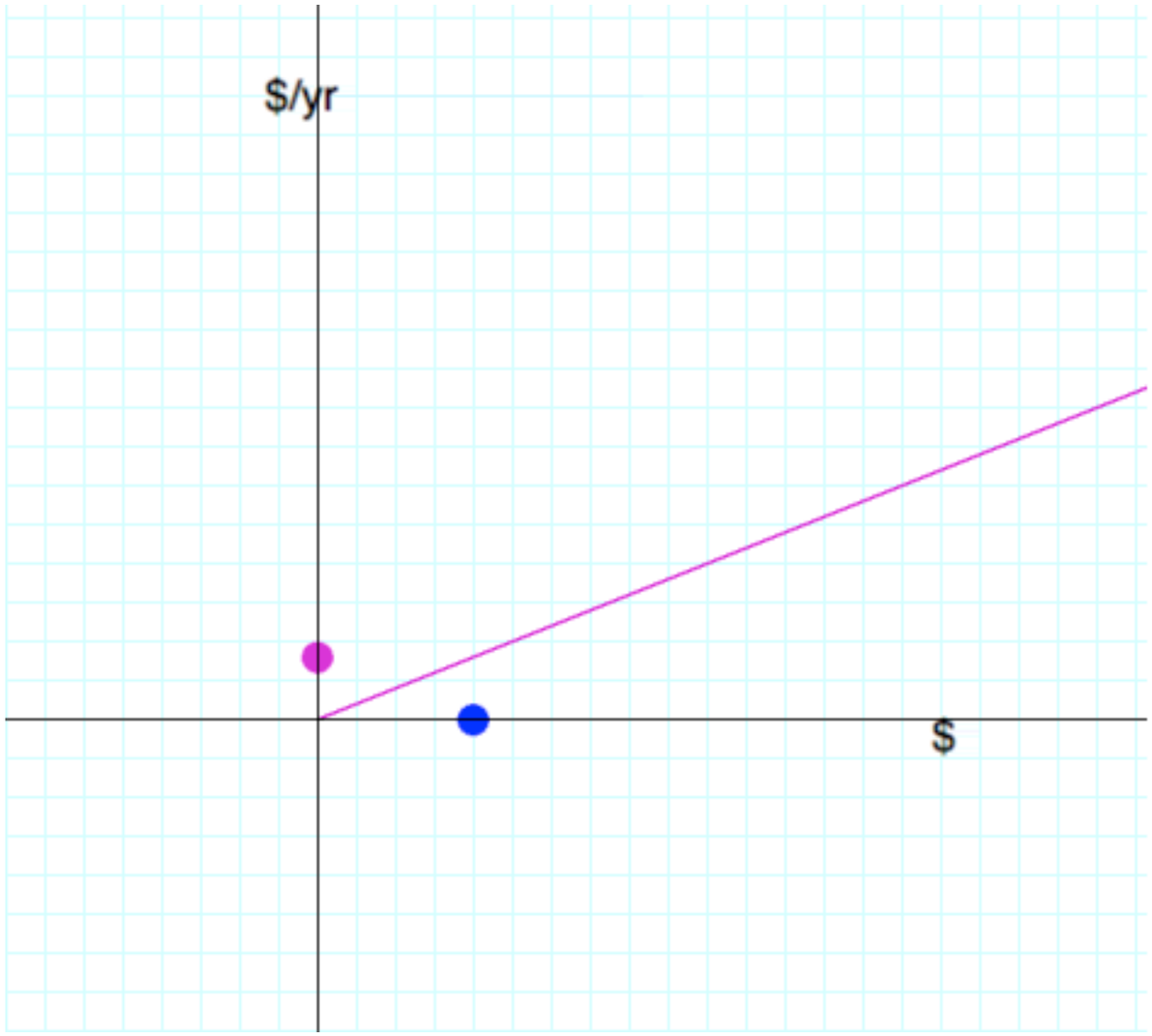
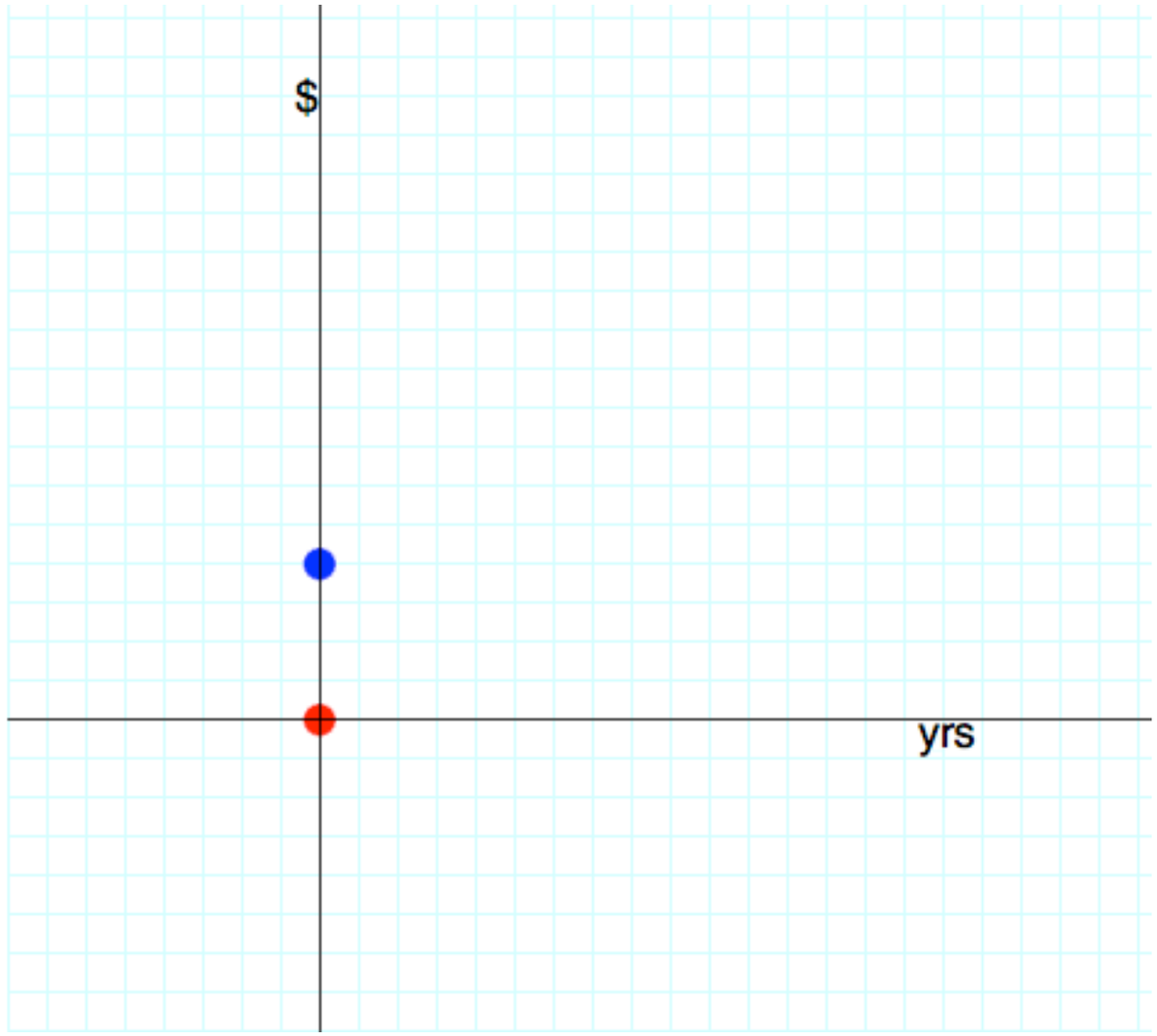


Tiffany: I'm not quite sure how jagged it would look, but like it'd go from this point, then you're going taking that and going up, and taking that and going up, kinda thing.

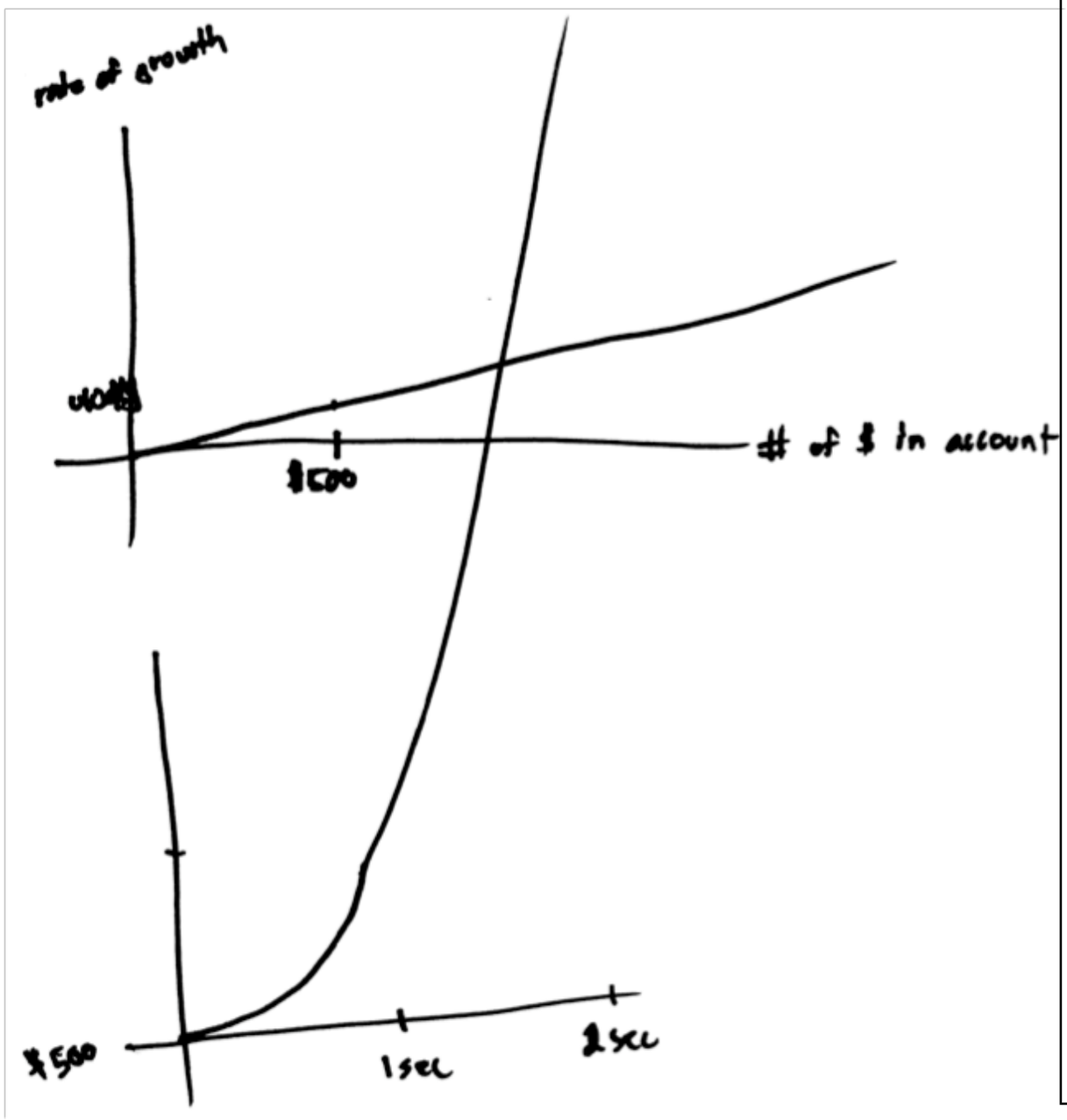
Derek

Derek





Derek



Carlos: So can you show me how the money in your- in your account is growing, umm.

Derek: On that axis?

Carlos: By moving your finger along this axis, yeah.

Derek: Like starts slow and then just keeps getting faster and faster.

Carlos: OK, umm, and what about the rate of growth?

Derek: It would also start slow and keep getting faster and faster.

Alternate Method

- http://www.math.ksu.edu/~cwcg/demo/rate_proportional_to_amount/studio/