

**Average rate of change**

# Assignment

Think back to our breakout on Thursday. What does it mean to have a constant rate of change?

1. How do you calculate an average rate of change?
2. What does average rate of change mean? (Note, this is different from question 2).
3. What is the difference between questions 2 and 3?
4. What does instantaneous rate of change mean? (**not** "how do you calculate it.")
5. What is a problem with looking at average rates of change over intervals of 1?
6. If you take the previous sections seriously, there are serious incompatibilities with the sections on arithmetic and geometric sequences. What are some of the problems with using arithmetic and geometric sequences to teach rate of change?

# Constant Rate of Change

1. What does it mean to have a constant rate of change?

# Constant Rate of Change

- $y$  has a constant rate of change with respect to  $x$  iff
  - $y$  and  $x$  are ***changing continuously***
  - For all  $\Delta x$ ,  $\Delta y = m\Delta x$ ,  $m$  constant.

# Average rate of change

2. How do you calculate an average rate of change?

# Average Rate of change

- For two data points  $(x_1, y_1)$  and  $(x_2, y_2)$
- average rate of change =  $y_2 - y_1 / (x_2 - x_1)$

# Average Rate of change

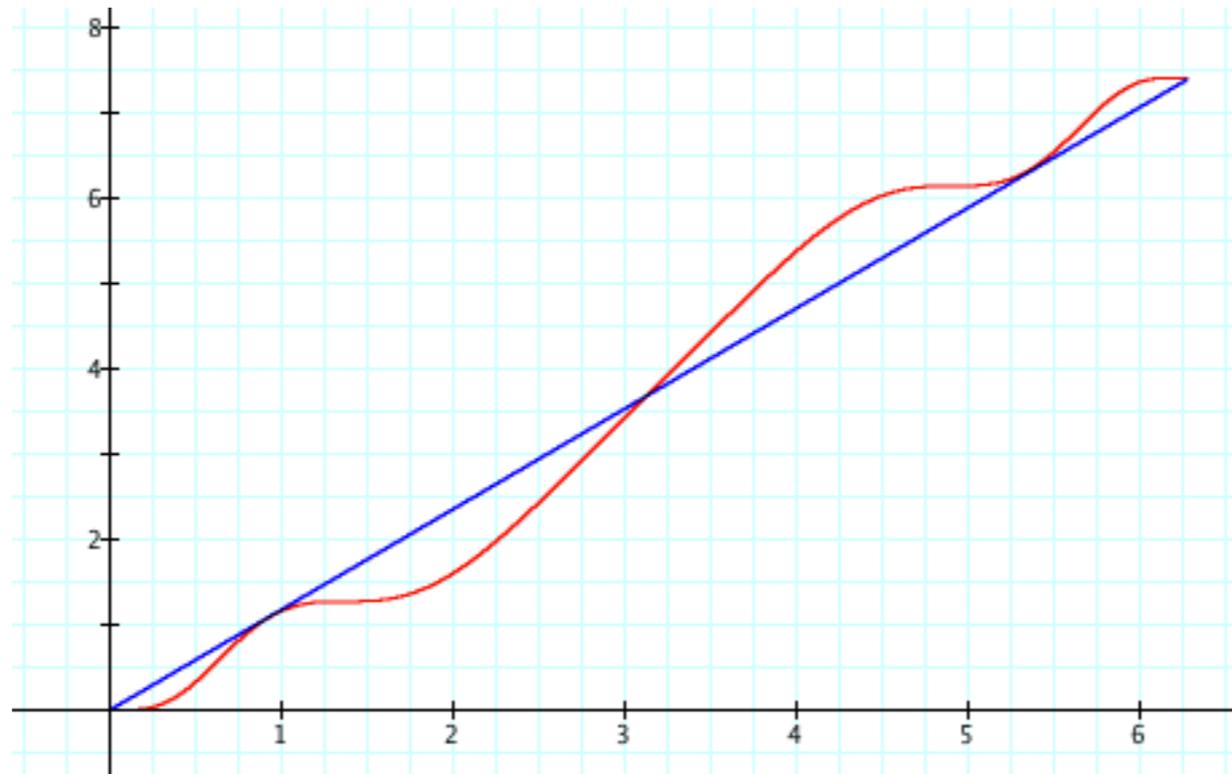
3. What does average rate of change mean?

# A trip

- I drive from work to home. Along the way, I have to stop for stop lights, I speed up and a slow down because of traffic, I spend a few minutes on the highway going really fast. At the end I have to slow down and find a place to park. When I get home, my average speed was 18 miles per hour (this is actually true)
- What does it mean that my average speed was 18 mph?
- How would your students answer this question?

# Average Rate of change

- The constant rate of change needed to start and end at the endpoints of two coordinated intervals.
- Average speed: The constant speed I would have to go to start in the same time and place and end at the same time and place as my actual trip.
- Slope of the secant line



# Instantaneous rate of change?

5. What does instantaneous rate of change mean?

# Instantaneous Rate of Change

- When the Discovery space shuttle is launched, its speed increases continually until its booster engines separate from the shuttle. During the time it is continually speeding up, the shuttle is never moving at a constant speed. What, then, would it mean to say that at precisely 2.15823 seconds after launch the shuttle is traveling at precisely 183.8964 miles per hour?

# Calculus Students' Answers

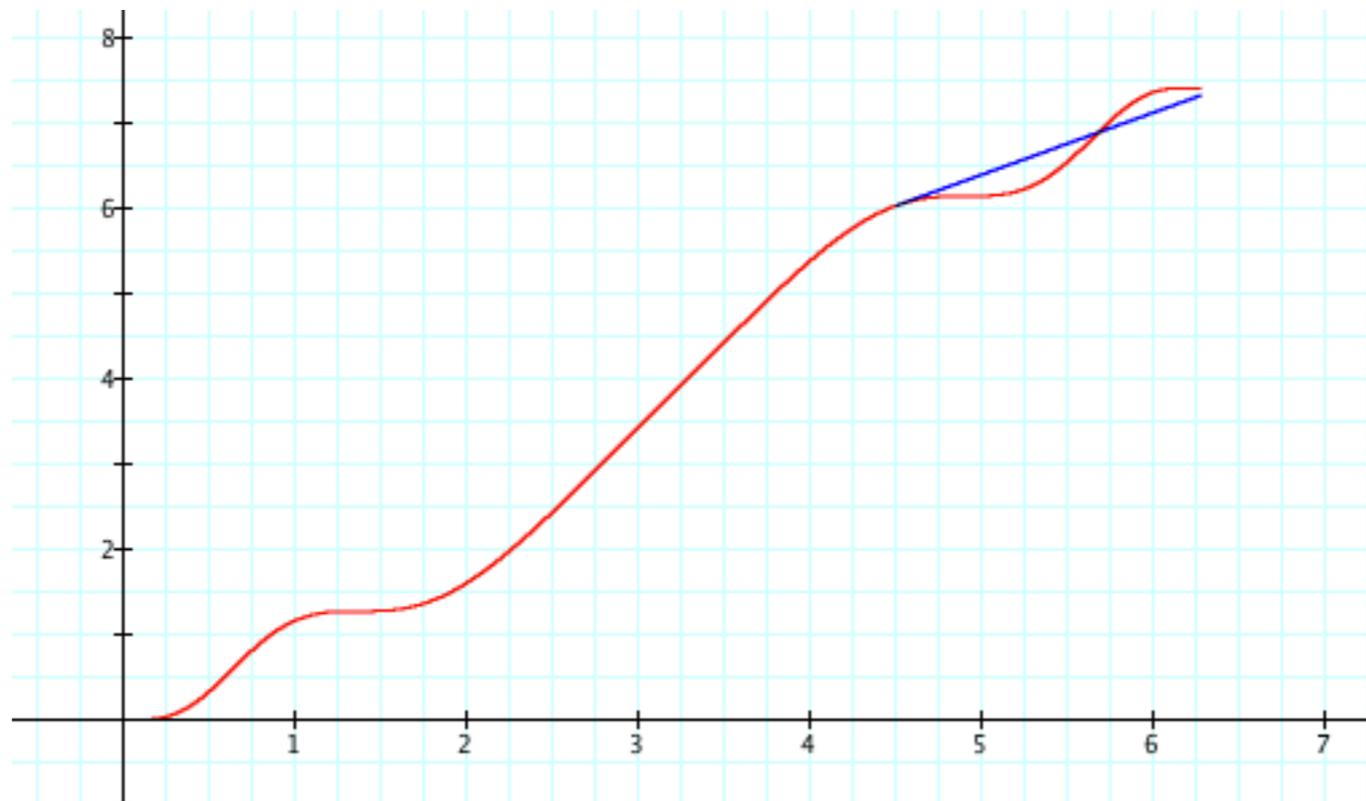
- This means that if I have a stopwatch and a speedometer hooked up together onboard the space shuttle, and I could hit a button to take a reading of both at exactly the same time, AND the stopwatch started at the launch, if I hit the button 2.15823 seconds after launch, the speedometer on the space shuttle said we were going 183.8964 miles per hour.
- Not sure what type of answer I am supposed to give.
- Are you asking for its rate? I don't understand what you are asking for
- I don't like math problems that have a solution expressed in words. I call that English.

# Saturday Math's Club's Children's Answers

- Instant speed is a funny idea. You say it the same way you talk about constant speed or average speed, but you don't mean the same thing. It's kind of like saying, "If at exactly 2.15823 seconds after launch the shuttle stopped accelerating, it would continue moving at a constant speed of 183.8964 miles per hour.
- It means that if you were to find the shuttle's average speed from 2.158225 seconds to 2.158235 seconds, and round to four decimal places, you would get 183.8964 miles per hour.

# Instantaneous Rate of Change

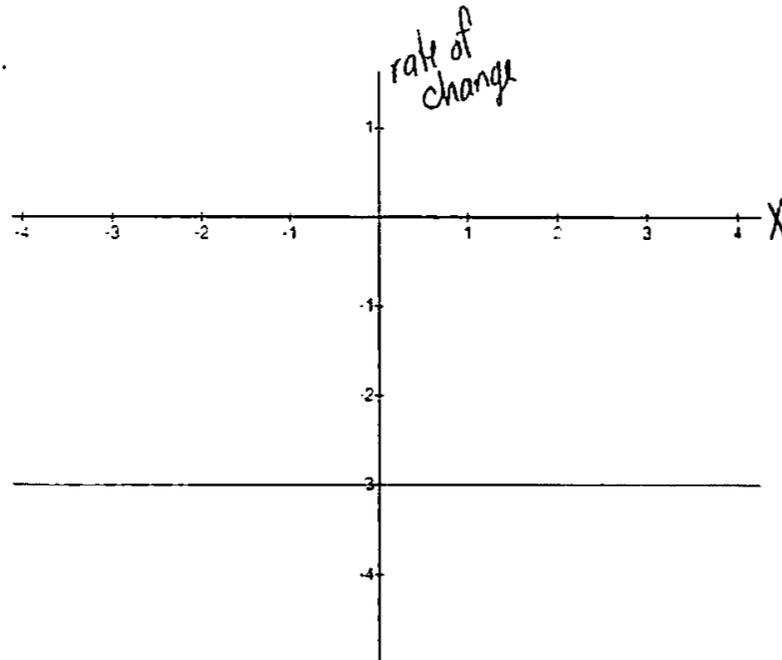
- Constant rate of change (if the rate of change stopped changing)
- Limit of average rate of change
- Slope of the tangent line



# Where does understanding rate take us?

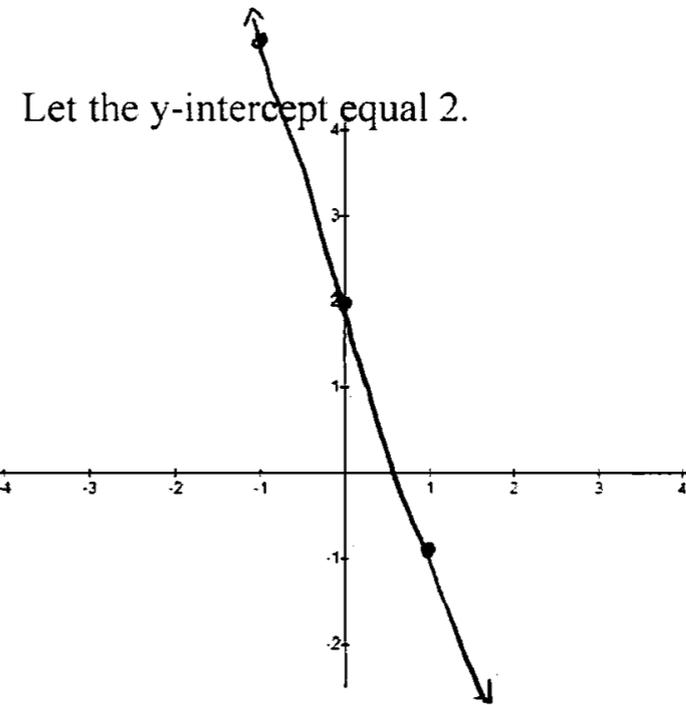
The following graphs relate the rate of change of a function to the  $x$  value of the function. Sketch a graph of the original function. Write the equation of the given line and your sketch.

1.



Equation

$$r = -3$$



Let the  $y$ -intercept equal 2.

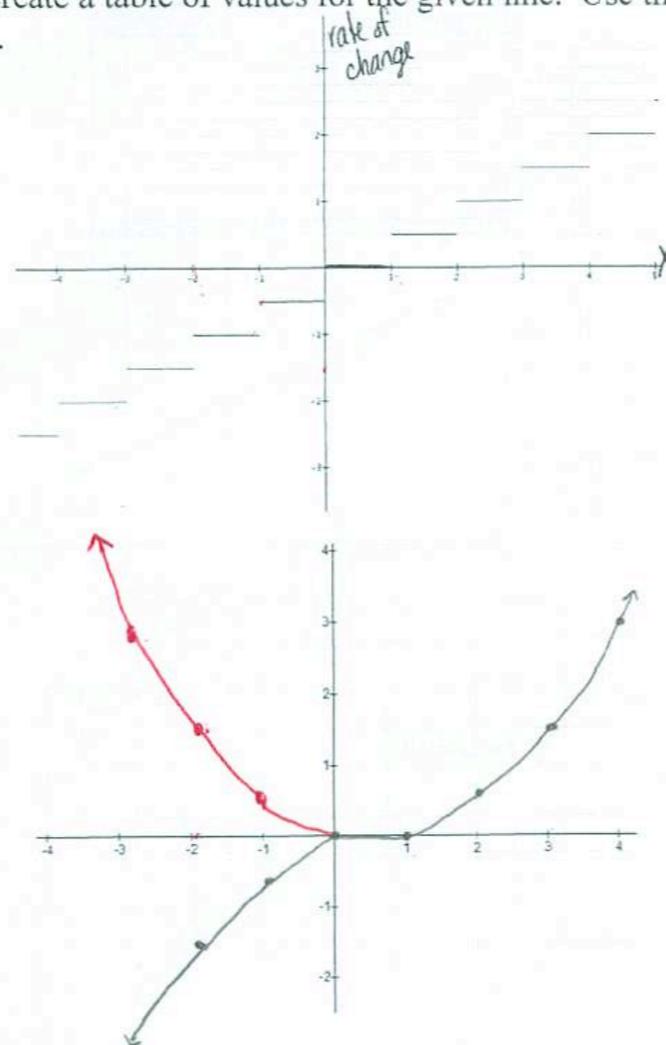
Equation

$$y = -3x + 2$$

# Where does understanding rate take us?

Below is a graph that relates the rate of change of a function to the  $x$  value of a function. Create a table of values for the given line. Use the table to sketch the original function.

4.



x value	rate of change
-4	-2
-3	-1.5
-2	-1
-1	-0.5
0	0
1	0.5
2	1
3	1.5
4	2

# Where does understanding rate take us?

